Lifelong Learning Programme

Le-Math

Le-MATH
Learning mathematics through new communication factors

MATHeatre Guidelines
for Teachers and Students

EN

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Le-Math theatre

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Learning mathematics through new communication factors

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Guidelines for MATHeatre Method

Teaching and learning mathematics through mathematical communication activities

Guidelines for Teachers and Students

Contribution for the preparation of these Guidelines

The Guidelines are the outcome of the collaborative work of all the Partners for the development of the Le-Math Project, namely the following:

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GENERAL REMARKS

Section G1

Introduction

In the proposal of the Le-Math Project, it is specified that a significant part of its outcomes is the preparation of Guidelines for Teachers that will help them develop their competences, in using theatrical activities as means that will enhance the teaching and learning of mathematics.

More specifically, these Guidelines concern the development of a methodology in teaching and learning mathematics through the creation of a tool called MATHeatre and which is to provide the basics for "Teaching and learning mathematics through math theatre activities".

The tool will have as final beneficiaries the teachers and pupils of mathematics at the school level, and it is expected to provide a framework that can be offered as an in-service training course to teachers who teach mathematics to pupils of age 9-18.

In the project Le-MATH, it is specified that this Method (MATHeatre) includes the development of sample teaching material and methodology for teaching Mathematics to 9-18 year old pupils, using specially designed theatre scenarios with mathematics as a main topic for following a direct or indirect approach. The method includes the development of guidelines that will help teachers to design and apply math theatre scenarios and organize a theatre festival or competition, in order to motivate pupils to learn, understand and appreciate mathematics. Developing communication skills and creativity is part of the methodology.

These Guidelines are providing a framework that will enhance the teachers' skills and prompt them to adopt a new teaching and learning tool. Through this tool,

pupils will follow a fresh and new approach and will be encouraged to communicate ideas, to comprehend various concepts and processes that have mathematical context, indulge in the philosophy and history of mathematics, reflect on the characteristics of the pioneers and to develop morals and aesthetic values that are inherent in the subject.

Through these Guidelines, the beneficiaries are expected to become acquainted with some aspects of the state of the Art in the area. Among these are included some essential elements concerning:

- What are the objectives of Mathematics and how can the MATHeatre approach help (or how can the MATHeatre approach be of value)?
- What are some fundamental aspects about the theoretical background concerning the exploitation of the MATHeatre as a learning approach?
- What are some models/approaches/examples in using MATHeatre activities as supporting means in learning/teaching?
- What could be the practice in integrating MATHeatre activities in teaching?

Furthermore, these Guidelines could be of value to teachers in Designing scenarios for teaching/learning. Among these, we would expect the development of competences on various subjects and activities like:

- The teacher or the students develop a scenario for a play based on mathematical ideas, aiming at improving motivation and communication skills in the context of the mathematical education of pupils.
- The teacher or the students develop/ adapt a scenario for a play based on an existing book, story, theatrical play or a related scenario in the area of the history of mathematics, mathematical concepts and pioneers. The aim is to increase the motivation, comprehension and the thinking process of the students and to improve their overall skills in the context of their mathematical education.

- The student develops a play or acts in a play using a scenario that will help in explaining a mathematical concept and presents it to his/her classmates.

- The students participate in a play or attend a theatrical performance that serves as a learning medium for understanding mathematical ideas, processes and concepts related to the educational value of a specific topic.

Through these Guidelines, it is expected to develop the competences of the teachers on **Implementing/Applying MATHeatre activities and scenarios for teaching/ learning**. These guidelines are expected to provide them with opportunities for addressing and discussing issues like:

- The recognition and the use (in the context of a usual math class or in the context of other activities, curricular or extracurricular) of MATHeatre activities/ scenarios/plays, aiming at improving motivation and various mathematical skills in the context of the mathematical education of pupils.

- The identification and use of MATHeatre activities and scenarios in the area of the history of mathematics, mathematical concepts and pioneers, aiming at introducing the students to a specific subject or in enriching their mathematical experience.

- The identification and use of MATHeatre activities/ scenarios/ plays, with the aim to help students to understand a mathematical concept, process or other ideas.

Finally, the Guidelines are expected to provide the teachers with valuable insights concerning the **Organisation and participation in festivals and competitions with MATHeatre activities**.

Section G2

What is the aim of MATHeatre?

The European Union identifies Mathematics as one of its cornerstones for development and in its targets for the Europe 2020 strategy outlines the importance of the promotion of this subject. Furthermore, it is useful to stress that among the targets of this strategy is to reduce the dropout rate from schools. So, any action that is contributing towards these targets is obviously a clear plus in the attainment of the objectives of EU.

In the proposal of the present project, it is explicitly claimed that MATHeatre aims on improving teaching and learning mathematics through math theatre activities.

In view of this, it is justifiable for someone to ponder: "on what grounds can we claim that there is evidence that this aim can be achieved?"

In the next few paragraphs, there are some arguments and considerations that lead to the adoption of the idea that this aim can be achieved. These arguments are based on the advantages and benefits of the theatrical activities in association to the goals and beliefs regarding the learning of mathematics, as well as to the principles that are supporting this learning approach. Furthermore, there are research findings that justify the claim that theatrical activities can contribute positively to this. Clearly, these associations are supporting the general feeling we have that theatrical activities can contribute positively in the learning of mathematics, as they can provide a forum for motivation, improve communication skills and promote problem solving abilities.

The Goals of Mathematics

The whole world gives emphasis to the mathematics education, by considering the goals and processes that are related to the subject. In nearly every country, there are efforts in employing interesting methods that will promote learning. For example, the Government of Alberta in Canada approaches the mathematics learning and teaching at the school level with a unique, creative and innovative way, by suggesting a broad set of characteristics related to active learning. We feel

that just a simple presentation of these ideas will give a concrete framework on what we are going to discuss later.

These are:

i. Beliefs about students and mathematics learning
Students learn by attaching meaning to what they do and they need to construct their own meaning in mathematics. Students of all levels benefit from working with a variety of materials, tools and contexts when constructing a meaning about new mathematical ideas.

The learning environment should value and respect the diversity of students' experiences and way of thinking so that they are comfortable in taking intellectual risks, asking questions and posing conjectures.

Students need to explore problem-solving situations, in order to develop personal strategies and become mathematically literate. They must realize that it is acceptable to solve problems in a variety of ways and that a variety of solutions may be acceptable.

Furthermore it is identified that:

ii. Goals for students
The main goals of mathematics education are to prepare students to:

- solve problems
- communicate and reason mathematically
- make connections between mathematics and its applications
- become mathematically literate
- appreciate and value mathematics
- make informed decisions as contributors to society.

Students who have met these goals:

- gain an understanding and appreciation of the role of mathematics in society
- exhibit a positive attitude toward mathematics

- engage and persevere in mathematical problem solving
- contribute to mathematical discussions
- take risks in performing mathematical tasks
- exhibit curiosity about mathematics and situations involving mathematics.

Teachers can assist students in attaining these goals by developing a classroom atmosphere that fosters conceptual understanding through:

- taking risks
- thinking and reflecting independently
- sharing and communicating mathematical understanding
- solving problems in individual and group projects
- pursuing greater understanding of mathematics
- appreciating the value of mathematics throughout history.

A significant role in the effective achievement of these goals can be attributed to the implementation and use of major mathematical processes. These processes are a critical aspect of learning, doing and understanding mathematics. Students must encounter these processes regularly as they learn mathematics, in order to receive a robust mathematics education. According to these principles students are expected to:

- use communication in order to learn and express their understanding
- make connections between mathematical ideas, mathematical concepts, everyday experiences and other disciplines
- demonstrate fluency with mental mathematics and estimation
- develop and apply new mathematical knowledge through problem solving
- develop mathematical reasoning
- select and use technology as a tool for learning and for solving problems
- develop visualization skills to assist in processing information, making connections and solving problems.

The consideration of these principles justifies the adoption of the theatrical approach as one of the means that can contribute effectively towards the learning of mathematics. This justification is further supported if we consider that indeed a theatrical approach is closely related to communication skills, problem solving skills, reasoning skills, etc.

Furthermore the American Alliance for theatre Education observes in their webpage that:

"Drama Improves Academic Performance''
Numerous studies have demonstrated a correlation between drama involvement and academic achievement. In addition to having higher standardized test scores than their peers who do not experience the arts, students who participate in drama often experience improved reading comprehension, maintain better attendance records, and stay more engaged in school than their fellow students. Schools with arts-integrated programs, even in low-income areas, report higher academic achievement.

Drama Students Outperform Non-Arts Peers on SAT Tests
The College Entrance Examination Board reported student scores from 2001, 2002, 2004, and 2005 using data from the Student Description Questionnaire indicating student involvement in various activities, including the arts. As compared to their peers with no arts coursework or involvement:

- *Students involved in drama performance scored an average of 65.5 points higher on the verbal component and 35.5 points higher in the math component of the SAT.*

- *Students who took courses in drama study or appreciation scored on average 55 points higher on verbal and 26 points higher on math than their non-arts classmates.*

- *In 2005, students involved in drama performance outscored the national average SAT score by 35 points on the verbal portion and 24 points on the math section.*

Attendance

Research indicates that involvement in the arts increases student engagement and encourages consistent attendance and that drop-out rates correlate with student levels of involvement in the arts.

- *Students considered to be at high risk for dropping out of high school cite drama and other art classes as their motivations for staying in school.*

- *Students who participate in the arts are 3 times more likely to win an award for school attendance than those who do not.*

The above arguments support the idea that the main aim of this project can be achieved. It is in this spirit that the present guidelines are setting a justifiable basis for promoting this notion.

PART A: METHODOLOGY - THE STATE OF THE ART

Section A1: Concrete Advantages for Teachers

Since ancient times, great mathematicians have used oratory skills to communicate their knowledge.

Through theatre, we aspire to do the same, as theatre makes it possible to construct this process of dissemination from scratch.

Also, staging concepts and characters, would allow students to understand better concepts that often seem abstract. For a mathematics teacher, incorporating theatre in his/her lessons is far from natural. Fears are logical. It is a question of changing the usual practice established in ordinary mathematics lessons. Even though the relationships between the teacher and the pupils are enriched, some worries are to be expected: from an academic role behind a desk, the teacher

becomes a director, and that is quite a difference! This guidebook is made to alleviate all these fears and to give teachers the desire to start this adventure!

The idea is not to make the teachers to completely change their way of teaching, but to open the door on integrating, once in a while, theatre activities in some of their sessions or workshops.

The primary technical theatre notions will be explained in order to carry out the different projects.

Advantages

The advantages of the introduction of drama into teaching practices are numerous. Indeed, theatrical techniques are often used in special pedagogical or socio-cultural situations such as the learning of a foreign language, for personal development, or to increase the enthusiasm in a group. So, why not in a mathematics session?

The universality of mathematics allows every mathematics teacher to use such a method as a tool to enrich their teaching. Our objective in this guidebook is to present a methodology that is easy to implement, whenever you want to introduce theatrical activities into your sessions or workshops.

The participants of the “Le-Math Theatre competition” will have to follow this methodology to compete.

You will be able to write your own scripts or use those already written as included.

Also included, are the criteria to test students in such activities and the criteria for the competition.

Using theatre in mathematics presents many interesting challenges for the class. The teacher is the stage director!

In this way, we can create a dynamic in the group where each student can exchange ideas, give input, listen and share, with the pleasure of working together.

Each student can also develop a socio-cultural awareness, autonomy, an open mind, imagination, creativity and self-discovery with the help of the teacher and learn to cope with the experience of performing in public and improving their self-confidence and self-expression.

Theatre reinforces the notion of sharing. The acceptance of authority is integrated with a playful frame.

Given instructions are more readily accepted, as becoming a theatre stage director has a real impact on the students. “Silence! On stage!” is effective, just try!

The creation of communication situations and a real interactivity (initial preparatory work in the class, rehearsals, final production, the performance itself and the discussions that follow the activity) around a mathematical theme, can be practiced in a theatrical context.

Students will learn to bring out, unlock and improve the fluidity of their speech, to develop their memorisation skills and to think and reason by using the language of mathematics, thus rendering this subject less “foreign”.

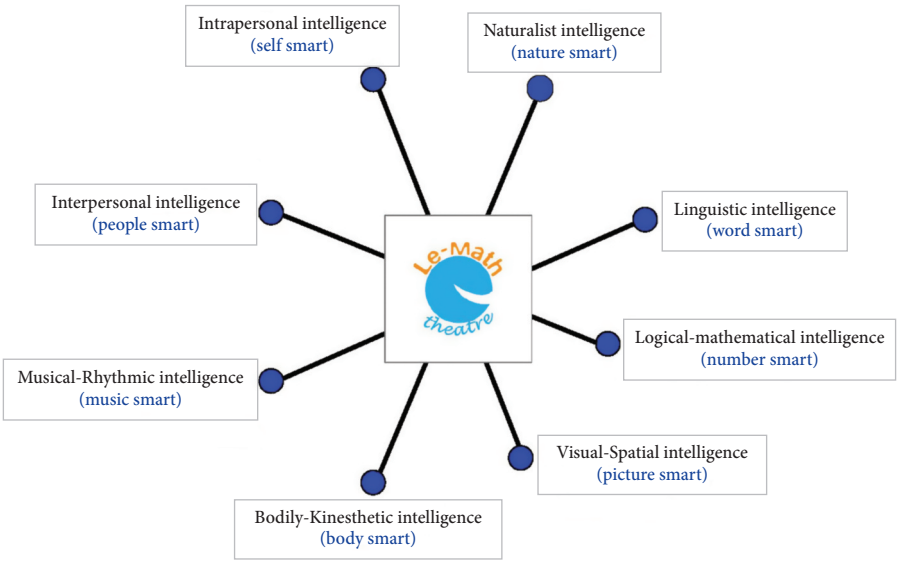
This approach will help to consolidate learning activities, as it enables working on rhythm, melody and intonation, sounds, tonalities and learning in general. Especially for the younger children, there will be improvement in their attention span, concentration and ability to listen.

Theatre is an art which combines, among others, music, dance, comedy, and leads to the discovery of related jobs such as sound control, lighting, set building, costumes and make-up.

Multiple intelligences and Drama

In 1983, Howard Gardner, Professor at Harvard University, released his book "Frames of mind" in which he developed his theory of multiple intelligences.

He suggests that each person has several types of intelligence, for which he naturally display’s more or less competence. There are eight of them:



Traditional mathematics lessons bring logical-mathematic intelligence into play, such as the ability to reason in the geometrical or numerical area, as well the ability to calculate and handle figures, numbers, and geometrical shapes.

The other forms or types of intelligence are often casted aside or are completely forgotten. However, by using this theory, which is often disregarded by teachers , it would allow for a better understanding on low-achievers and cultivate the desire to invest more on the positive stimulation of their intrinsic motivation.

Pedagogically, mixing drama and mathematics allows us to solicit almost all the different types of intelligence:

- **Logical-mathematics**: The mathematical content, worked in the classroom and processed in the play, might be reinforced after the theatrical activities. Moreover, these skills are also required in the elaboration of the script, the play.

- **Spatial**: Recognition of the notion of space in the staging of the play. The movement of students themselves during the play and the recognition of their own position in the space, as well as the position of their fellow students.

- **Kinaesthetic**: When students are acting, they represent a character or a mathematical symbol. The notion is imprinted in their minds through the movements enacted by their own bodies.

- **Linguistic**: The work begins with the writing of a script or with the study of a script. In all cases, the language is a method of communication and therefore must be worked, adapted to the audience and perfected, as it is the basis of the play.

- **Interpersonal**: The relationship between the student and the teacher. The discussions between the students during the development of the script, the elaboration of the play, the feedback of the activity and the work in a group in general, improve communication.

- **Intrapersonal**: The student, as an individual, has to contemplate on the notion studied in order to understand it. Memorizing their text and conducting individual rehearsals, before those of the group can be of great help.

- **Musical**: Music smarts can be cultivated in a musical, or if there is music or songs in the play. Moreover, during the play, musicality is present in the modulations of the voice, the volume, the rhythm and the speed of speech, which are necessary for the clarity and the pleasantness of the play.

- **Naturalist**: The decor can make students imagine they are in meadows, near the sea or in a forest. Everything is in their imagination and theatre allows that.

Moreover, even more important ... the pleasure, the game!

The entertainment aspect is promoted to lessen the effect of the restraints of learning. The first and foremost interest has to be pleasure and entertainment and not learning.

The pleasure they will have while acting in theatre plays or activities, will increase their intrinsic motivation to learn, to memorize and vice-versa. (Nicolaidou & Philippou, 2003). Additionally, they will develop a tendency to be more persistent in solving mathematical problems (Lepper & Henderlong, 2000).

Theatre activities create a state of relaxed concentration, thus encouraging better learning.

However, we have to keep in mind that drama is not a miraculous solution, but mostly serves as a joyful and artistic tool for learning, which could have an important impact on the mathematical achievements of the student.

This is the reason why teachers need to take into account the following points in order to succeed:

How to deal with the heterogeneity of groups?
Most pupils are happy to practice theatrical activities. However, it may happen that the enthusiasm is not shared by everyone.

There are two types of students (extrovert or introvert) to take into account: the students who are failing in maths or who have been suffering from a long-term blockage on the subject for a long time or are simply demotivated by the subject; then there are the successful students who are interested or who have a gift for mathematics.

For the first group, this new method will enable them to approach mathematics in a fun way, using other personal qualities rather than their mathematical potential according to multiple intelligences.

The second group may not see the point of this new approach, as they are already successful in that subject. There is a risk of having a negative attitude towards this new practice, and they even might be hostile. However, they need to be convinced that the usefulness of this method is to be able to express mathematical notions, verbalise them, explore them in an entirely different way and relate these notions to the audience, elevating the basis of learning to greater heights.

In both cases, pleasure is the mean to an end (Multiple Intelligences: inter-personal and team work at all stages). The pleasure to share with the other students, to learn together (rehearsals) and in the end to perform together (the play), creates a strong link between the students and the teachers.

There may be students who still resist this technique: students who do not like theatre, who are too shy or have other reasons for refusing to be actors, like the fear of being ridiculed or judged.

They can still be involved in other important roles and bring out their strengths while offering technical support, writing, directing and while helping with the costumes, the set, the make-up, etc.

How to reassure the most reluctant teachers?
The aim of such activities is not to be carried out continuously throughout the year, but maybe once or in a workshop where you are not confined by the curricula.

Sometimes, teachers are afraid of losing precious teaching time and that they will not be able to grade their students. In this guidebook, you'll find criteria that will help you to evaluate your students if needed, so that time can be spent to work on something else.

Furthermore, some countries have now changed their method of evaluation: teachers evaluate the students' knowledge and skills, according to the “common core of knowledge and skills”, therefore they also need to have some information that is often difficult to observe with traditional sessions. This kind of activity allows them to evaluate the students more efficiently, e.g. social and civic skills, initiative, autonomy, etc.

Some teachers might also be worried about their own lack of training or experience in these practices: The fear of not being able to maintain the coherence between mathematical lessons, support lessons, learning objectives and theatre activities, along with the fear of getting out of the classical frame of teaching or losing the role of being in charge can prove stressful for the educator.

There are worries about dealing with the whole concept of theatre. However, it is not necessary to perfectly master theatre to use its techniques. It is important though, to be able to manage a group, and the problems associated with it, and in this aspect teachers are able to perform well!

Teachers might also face problems different from those they usually encounter in the classroom, like too much noise, disturbances, excitement from the students - especially from the younger ones. There may be difficulties in managing noise levels, and new tactics need to be deployed. Some drama trainers use the sign language, raising and waving hands to avoid the noise in the classroom. It should be noted though that applause is an important part of the play, as it is a way for the audience to interact.

Teachers have to calm the most boisterous learners and to encourage those that are more timid. Furthermore, they need be sure of their ability to realize such a project for the best interest of everyone involved.

So what qualifications are needed to begin this drama practice?

It is certainly an advantage if the trainer had an experience in theatre, but it is not necessarily a requirement. Most people have seen at least one play or have read a script.

It is not so difficult for teachers to become actors or stage directors, as in a sense they emulate the experience when they enter the classroom. They have their public, and they must convince their audience using rhetoric, body language, etc. Just like the way that famous mathematicians, thinkers or philosophers have done for centuries in the past.

The role of the teacher is to create a fun atmosphere beneficial to the game, to reassure the learners and to encourage their participation. The teacher needs to instil a sense of mutual respect, to establish a non-judgemental atmosphere where humility and collectivism are necessary, as well as allowing imagination to thrive.

From the most reluctant to the most motivated, let's now look how we can integrate theatre into our mathematical practices and lead our students to increase their comprehension and interest for the subject.

Different types of theatrical activities

It is possible to set up a theatrical activity in the mathematics lesson in different ways depending on the objectives, but also depending on the number of sessions the teacher chooses to use for the work.

A theatrical activity setting linear equations
(Collège Saint-Charles, Guipavas, France)

To discover a new concept

The setting of a theatrical activity can help the teacher to introduce a new concept. In this way, the teacher can prepare a discovery activity that will allow students to become familiar with new content.

Role plays are appropriate to explain mathematical methods where each student has a specific role in the game, e.g. exploring proper or improper fractions and mixed numbers, simplifying fractions (Pope S., 2012), or solving equations (Muniglia M., 1994).

Gerofsky (2011) claims that “the whole group improvisation drama in an «as if» setting engages students through immersive emotional and contextual modes of understanding“.

Using theatre to teach mathematics involves acting, which is qualitatively similar to children's spontaneous role-playing. Pallascio and Lajoie (2001) study role-playing as an efficient tool for making students active in a given situation.

The objective of theatrical activities, similarly to role-playing, when used in teaching contexts, is to lead student-actors and other student spectators to learn something from the given situation. When dramatizing a mathematical concept, pupils use facial expressions, role-playing, improvisation etc. They work in groups and improve their understanding of mathematics through writing scripts and acting.

The activity is conducted before the lectures. Its length is relatively short.

A theatrical activity about the algorithm of simplifying fractions

To deepen a concept

The use of theatrical activity could also be used after studying a concept following the theory and the classical training exercises. Acting in a play or writing a script is a good way to master a concept. The teacher must decide how much time should be given for the activity.

Setting a short activity

The teacher can choose an activity for a sketch. This can be done at the end of a work session. The sketch can use a small number of students and takes place in the classroom, and needs few or no specific materials. It focuses on a single concept.

The teacher can ask the students to represent, in a role-playing game, the different notions studied. Students can work in little groups and set up a little sketch in class. This activity can be useful for the students, as it can be used to check their comprehension.

"The legend of number 10", Colegiul National Coriolan Brediceanu, Romania, 1st place, category 9-13 in MATHeatre competition 2014

Setting a longer activity

A theatre play is an excellent way to master a concept. The teacher can organize an annual or biannual project. The activity can take place during class or during a workshop outside school or class hours. One or more sessions per week can be made available for the drama workshop. The focus could be on a broader mathematical content. The theme of the play can be the story of one mathematician (or more), and the pretext-to develop mathematical discoveries-

combining several concepts studied during the year, and could be an excellent opportunity to work along with other disciplines, e.g., PE, language ... This would allow students to reactivate their knowledge and to reinvent their ability to synthesise. At the end of the year, the show could be put on, as a reward for their dedication.

According to Martin Andler, March 2014, some solutions to fight the declining success of students in mathematics, the lack of motivation for the subject and the cut in staff in the mathematics sectors(PISA 2012 results, What students know and can do-OCDE), are to make the mathematical sessions less theoretical and abstract. This can also help to give meaning to students' learning, to change their usually passive behaviour to an active one, to work in groups, to work in multi-disciplines, to put mathematics in perspective by contact with research, real-life applications, but also with Art. Through art, emotions can be felt and expressed, and they can facilitate the process of learning by helping to activate the long-term memory. Furthermore, art can open new paths for the students and help them pursue their desires, their destiny, and to develop their autonomy.

The MATHeatre project has many of these assets and could give students an alternative approach in learning mathematics, by increasing their intrinsic motivation, allowing them to feel involved in their learning, and above all, change their way of thinking about traditional mathematical sessions.

Section A2: Setting Goals and Objectives for Learning

The question of motivating students has become a leading concern for mathematics educators in many countries. The motivation of students becomes especially relevant to mathematics education in Europe in light of recurring challenges on how to contribute towards the European Union's vision in achieving a high economic and scientific development.

The use of Matheatre can be viewed as a challenge to increase the students' intrinsic motivation. The use of the method needs an appropriately differentiated curriculum designed to address the individual characteristics, needs, abilities, and interests of different groups of students. Motivation and positive attitude towards Mathematics in general is a kind of internal drive that leads students to pursue a course of action. There has been extensive research on the role of attitudes and motivation in learning mathematics. The findings show that positive attitudes and motivation are related to success in learning. Unfortunately, the research cannot indicate precisely how motivation affects learning. That means we do not know whether it is the motivation that produces successful learning or if it is the successful learning that enhances motivation.

Although mathematics is considered a strand in the theory of intelligence; (Gardner, 1999; Sternberg, 1985) research have demonstrated the need for students to have access to advanced mathematical content (Johnson & Sher, 1997) and exposure to authentic and challenging mathematics problems (Johnson, 1993; Kolitch & Brody, 1992).

However, mathematics curricula and didactical approach are often inappropriate because of the highly repetitive nature of the courses and their lack of depth (Johnson & Sher, 1997; Kolitch & Brody, 1992; Park, 1989; Westberg et al., 1993). Thus, there is a strong need for research about the kinds of educational experiences that should be provided for the deeper involvement of students. A participatory teaching process, as well as research on the use of technological tools could effectively and appropriately enhance the instruction.

- The mathematics curriculum including MATHeatre method should bring teachers and students to work collaboratively (Tomlinson et al., 1995). Students will benefit greatly, both academically and emotionally, from this type of experience. They will learn from each other, reinforce each other, and help each other over difficulties. Talented students learn better in a nurturing, emotionally safe, student-centred environment that encourages inquiry and independence, and connects the school experience with the world at large. Less talented students benefit as well, because the cooperative learning form could lead to a change of attitude towards mathematics; the understanding will be easier, deeper, as they will see the inside of a mathematical problem, they will become part of the problem; they will participate in the solution; they will be first emotionally, then intellectually involved with the mathematical content of lessons.

- The mathematics curriculum should stress mathematical reasoning and develop independent exploratory skills (Niederer & Irwin, 2001). For instance, this is exemplified by using problem solving and discovery learning, engaging in special mathematical projects, discovering formulas, looking for patterns, and organizing data to find relations. Activities should help students to develop structured and unstructured inquiry, reinforce categorization and synthesis skills, develop efficient study habits, and encourage engagement in divergent questions.

- The mathematics curriculum including MATHeatre should be flexible (on the basis of an assessment of students' knowledge and skills). Curricula pieces containing the mathematical communication tools developed by LE-MATH project should promote self-initiated and self-directed learning and growth. Content and learning experiences can be modified through acceleration, compacting, variety, reorganization, flexible pacing, and the use of more advanced or complex concepts, abstractions, and materials.

- Inquiry-based, discovery-learning approaches that emphasize open-ended problems with multiple solutions or multiple paths to solutions are extremely effective. Students can design their own ways to find the answers to complex questions. An effective instructional technique for open minded students that promote self-initiated and self-directed learning is the use of a-didactic situations. In the "Theory of the Situations" by G. Brousseau (1997), the a-didactic situations have three phases: phase of action, phase of formulation and stage of validation.

The phase of action corresponds to mathematics in reality and consists of making proper the decisive strategies in a situation of concreteness. The phase of formulation consists of finding a code of communication for the strategy being used. Finally, the situation of validation is that in which the participants decide who came up with the optimal strategy. In order to answer this question, the students have to formulate "theorems in action" that allow the optimisation of possible solutions. Thus, from a pedagogic point of view, the "game" assumes a crucial role. The student learns to move from the phase of action to the public negotiation (in class and without the direct intervention of the teacher) of all the possible strategies (the theorems in action). The teacher prepares the a-didactic's situation and remains arbiter of the rules that need to be respected. All the phases are directly managed by the students.

Curriculum in European Schools

The national curricula of each country set the legal requirements for teaching and learning mathematics, and provide information to help teachers implement mathematics in their school. The national curriculum lies at the heart of our policies to raise standards. It sets out a clear, full and statutory entitlement for learning. It determines the context of what will be taught, and sets attainment targets for learning. It also determines how performance will be assessed and reported. An effective national curriculum, therefore, gives teachers, pupils, parents, employers and the wider community a clear and shared understanding of the skills and knowledge that young people will gain at school. It allows schools to meet the individual learning needs of the pupils and to develop a distinctive character and ethos rooted in their local communities. It provides a framework, within which all partners in education can support young people on the road of further learning. Getting the National Curriculum right presents difficult choices and balances.

It must be robust enough to define and defend the core of knowledge and cultural experience that is the entitlement of every pupil and at the same time flexible enough in giving teachers the scope to build their teaching around it in ways that will enhance its delivery to their pupils. The focus of this National Curriculum, together with the wider school curriculum, is therefore to ensure that pupils develop from an early age the essential literacy and numeric skills they need to

learn; to provide them with a guaranteed, full and rounded entitlement to learning; to foster their creativity, and to give teachers discretion to find the best ways to impart pupils a lifelong joy and commitment for learning.

Every European country has developed its own curriculum based more or less on the principles of the National Curriculum. Below we try to give as many details as we can about the mathematical topics and set them in opposition with the application of MATHeatre methods, as they given or found by the participating countries in the project.

Section A3: Putting New Theories into New Practices

Mathematics is a form of reasoning. Mathematical thinking consists of thinking in a logical manner, formulating and testing conjectures, making sense of things, and forming and justifying judgments, inferences, and conclusions. We demonstrate mathematical behaviour when we recognize and describe patterns, construct physical and conceptual models of phenomena, create symbol systems to help us represent, manipulate, and reflect on ideas, and invent procedures to solve problems (Battista, 1999).

Mathematics during the past decades became a lesson where students had to memorize formulas, apply the formulas in order to get a numeric result, and solve a large number of numeric exercises. If a student was able to learn the algorithm and apply that algorithm then that student was considered to be successful. Critical thinking was set aside, and communication in mathematics was limited to teacher instructions. In order for this attitude to change, new practices need to be imported in mathematics schools.

The expression “best practice” was originally borrowed from the professions of medicine, law, and architecture, where “good practice” or “best practice” are everyday phrases used to describe solid, reputable, state-of-the art work in the field. If a professional is following best practice standards, he or she is aware of current research and consistently offers clients the full benefits of the latest knowledge, technology, and procedures. If a doctor, for example, does not follow contemporary standards of medicine and a case turns out badly, peers may criticize his decisions and treatments by saying something like, “that was simply not best practice” (Zemelman, Daniels, Hyde, 2005).

The poor performance of U.S. students in math can be traced to the method used to teach math at the elementary level. The focus is on specific problems and not on building the foundations necessary for understanding higher level math. These foundations can only be built with a mathematics program that teaches concepts and skills, and problem-solving (Daro, 2006).

The reform movement in mathematics education can be traced to the mid-1980's and was a response to the failure of traditional teaching methods, the impact of technology on curriculum and the emergence of new approaches to the scientific study of how mathematics is learned. Basic to the reform movement was a standards-based approach to the "what and how" of mathematics teaching (Battista, 1999).

In the new mathematics, the focus is on problem solving, mathematical reasoning, justifying ideas, making sense of complex situations and independently learning new ideas. Students must be provided with opportunities to solve complex problems, formulate and test mathematical ideas and draw conclusions. Students must be able read, write and discuss mathematics, use demonstrations, drawings and real-world objects, and participate in formal mathematical and logical arguments (Battista, 1999). The process standards are organized around the areas of problem solving, reasoning and proof, communication, connections and representations (National Council of Teachers of Mathematics, 2000).

A set of basic assumptions about teaching and schooling practices is implicit in this reform movement. First, all students must have an opportunity to learn new mathematics. Second, all students have the capacity to learn more mathematics than we have traditionally assumed. Third, new applications and changes in technology have changed the instructional importance of some mathematics concepts. Fourth, new instructional environments can be created through the use of technological tools. Fifth, meaningful mathematics learning is a product of purposeful engagement and interaction which builds on prior experience (Romberg, 2000).

In order for the students to change their attitude towards mathematics, practical teaching has to take place.

Essential characteristics of an effective standards-based mathematics classroom include:

- Lessons designed to address specific standards-based concepts or skills
- Student-centered learning activities

- Inquiry and problem solving focused lessons
- Critical thinking and knowledge application skills
- Adequate time, space, and materials to complete tasks.
- Varied, continuous assessment, designed to evaluate both student progress and teacher effectiveness (Teaching Today, 2005a).

The implementation of a standards-based math curriculum brings with it some special challenges. In addition to ensuring students are actively engaged, teachers should adhere to the following guidelines:

- Create a safe environment where students feel comfortable
- Establish clear procedures and routines
- Provide both challenge and support
- Use carefully assigned and well-managed cooperative groups
- Make frequent real life connections
- Use an integrated curriculum
- Provide engaging educational experiences that are relevant to students
- Present activities where students produce and share products (Teaching Today, 2005b).

The goal of teaching mathematics is to help all students to understand concepts and use them powerfully. Students should develop true understanding of mathematical concepts and procedures. They must come to see and believe that mathematics makes sense, that it is understandable and useful to them. They can become more confident in their own use of mathematics. Teachers and students must come to recognize that mathematical thinking is part of everyone's mental ability, and not confined to just a gifted few (Zemelman, Daniels, Hyde, 2005).

Research for many years has shown that understanding the way mathematics works increases the ability to learn, to remember, and to apply mathematics.

Five intertwined processes build mathematical understanding. Teaching for conceptual understanding means helping students build a web of interconnected ideas. Teachers provide experiences for students in which they actively engage in these essential processes:

- making connections
- creating representations
- using reasoning and developing proofs
- communicating ideas
- problem solving (Zemelman, Daniels, Hyde, 2005).

Student mathematics achievement will improve if teachers consistently use research-based instructional practices to develop both computational fluency and a deep understanding of mathematics concepts by engaging all students consistently and effectively in the following mathematical practices:

- Providing Explanations – Students explain how they think about the meanings of ideas and the mathematical reasoning they use to make sense of calculations, problems, and/or ideas.

- Making Justifications – Students use mathematical reasoning (both inductive and deductive) to justify why their own or others' ideas are or are not valid/accurate. They identify relevant and age-appropriate mathematical definitions, properties, processes, counter examples, and/or established generalizations to present a robust logical argument and demonstrate precision.

- Formulating Conjectures & Generalizations – Students make and test conjectures and generalizations about the application of their own and others' mathematical ideas and processes to the general case, special cases, and/or different contexts.

- Using Multiple Representations – Students make, use, and connect multiple mathematical representations – equations, verbal descriptions, graphs, concrete models, charts, tables, everyday life situations, and diagrams – to "mathematize," make sense of, solve, and/or communicate about the questions, quantities and relationships in problems and ideas.

- Engaging in Metacognition – Students practice mathematical metacognition by reflecting about:
  - what/how they think about a math idea or problem
  - disequilibrium, breakthroughs, and "stuck-points" in their thinking
  - ways their mathematical understanding is developing
  - specific ideas or learning episodes that influenced their thinking.

- Making Connections – Students make and discuss connections between their prior understandings and the new mathematical concepts and skills they are learning, between their thinking and others' ideas, and between the mathematics they are learning and other contexts/content (Teachers Development Group, 2010).

A small theatre play of about 20 minutes, will make class more interesting and will enable greater learning for students. Students will be able to explain how they think about the meanings of ideas and the mathematical reasoning they used to understand them. For many students, studying math concepts for a long time makes them confused, especially if they are not able to follow all the algorithms. However, with the play they will be able to make connections between prior and new knowledge and also a connection of mathematics and real life situations. They will also create representations and by moving between these representations of mathematical concepts they will make connections between these concepts. Since making connections requires reasoning, students should be provided with such experiences.

Teachers need to ensure that students gain experience with a variety of strategies and are able to decide when to use each one. With the most powerful strategies, students create their own representations. The standard strategies of looking for a pattern and using logical reasoning are overarching and are essential in doing mathematics. Students must be encouraged to look for patterns and to use logical reasoning in every problem. But at a more specific level, students should develop capabilities with five critical strategies that are based on creating representations:

- Discuss the problem in small groups (language representations)
- Use manipulatives (concrete, physical representations)
- Act it out (representations and bodily-kinaesthetic sense)

- Draw a picture, diagram, or graph (visual, pictorial representations)
- Make a list or table (symbolic representations) (Zemelman, Daniels, Hyde, 2005).

In mathematics, students should be encouraged and helped to communicate their ideas by using a full range of language representations—speaking, writing, reading, and listening. Communication and reflection go hand in hand. Even though symbols are used to represent the most abstract aspects of mathematics, the symbols represent ideas that are developed and expressed through language. Oral language—discussing, verbalizing thoughts, "talking mathematics" for most students, most of the time, greatly facilitates their understanding (Zemelman, Daniels, Hyde, 2005).

MATHeatre also gives the opportunity to low-achieving Students (LAS) to become a member of the group and speak mathematics and also communicate mathematical ideas. Something that is impossible in a traditional classroom where a LAS is trying to spend his time with other activities such as drawing or playing with a smartphone.

In many countries in Europe and also Australia, US and more, many teachers have realized that something is wrong with the traditional classroom, and they are trying to apply new practices to their teaching. These new practices include:

- Mathematics Theatre
- Mathematics Competitions
- Mathematics Poster Design
- Mathematics Constructions
- Mathematics Art
- Dance
- Music
- Math Stories
- Math Scripts writing and so on. All these Practices develop creative learning in curriculum subjects and put the student in the centre of the activity.

Section A4: The Theatrical Approach

How to Make a Math Teacher or Student a Mini-Director

The theatrical approach requires a new perspective for the teacher and the students. It means that we have to consider the partners in the learning process as theatrical directors, writers of theatrical scenarios, actors or performers, stage managers, lighting and sound technicians and so on. Clearly, at this elementary level we are not going to consider all the details of the staff involved in a professional theatrical performance. However, it is useful to utilize as many of the activities, characteristics and facilities that are employed in a professional approach, so that we manage to achieve the aim of MATHeatre, as presented earlier. Below we present some points that will assist us in realizing the advantages of this approach.

The teacher-director: In this context the teacher, further to the formalities of the role of a director, he shares his knowledge and teaches. In this role the tutor, like a director listens and supports, shows and directs. His students actively participate and through constant communication, using every kind of means (language, figures, expressions of the face, etc.) they learn.

Nothing has changed from the ordinary class. Yet, one can imagine that by incorporating drama and performance in pedagogical practices we facilitate

learning, we provide means for communication, explanations and active involvement in the process of learning.

The teacher is in the classroom as a theatre director. He speaks, listens and organizes the lesson by taking a didactic approach to the discovery of knowledge and the understanding of concepts, processes, and methods.

The teacher-director will enable his students to become actively involved in their learning, to live and experience mathematics by implementing communication strategies through various theatrical devices that will help the students to better understand the concepts that are studied by performing in a virtual or real scene.

The teacher, thanks to drama, will help students to better communicate, share, confirm assumptions, arguments, experiences and to formalize mathematical concepts through theatrical activities.

The student will become the centre of his own learning; he will be the actor of his own cognitive process and gain confidence in himself through his personal involvement and his interaction with other classmates.

The learner will be guided in his mathematical learning, by developing both his knowledge and skills. Furthermore, he should be directed to enjoy the subject, realise its values and understand its importance through theatrical activities referring to historical developments and important figures of mathematics.

The theatrical activities are expected to encourage, promote, deepen and consolidate students' knowledge of the subject.

The teacher-director will ensure through theatrical activities that involve all the students, that everyone can express himself, find his place in the group and participate in the reflections, contemplations or realizations of the various mathematical concepts, processes or methodologies.

Each student will be able to feel directly concerned and involved in the learning process by participating or performing in a theatrical act in front of an audience.

The teacher will guarantee a safe working environment with mutual trust, constructive and fruitful relationships and exchanges.

Additionally, he will probably have to set up the "rules of the game" that each student will need to conform to in order to progress; like listening to others and respecting their points of view. A sort of modus vivendi that has to be accepted by everyone.

The students will learn to be attentive and caring towards their fellow co-actors and will learn to appreciate the hard work and the involvement of the members of their team.

The student actor will discover a new vision of mathematics with a commitment to his whole being, both physically and intellectually.

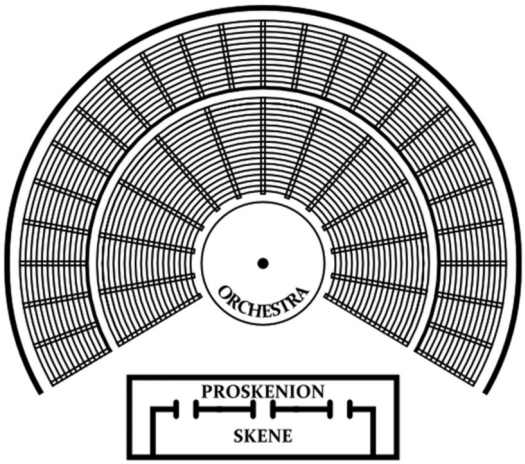
All this can be done within a different learning environment; with small exercises of improvisation, by reading aloud, with small games of diction, and with gestures and movements which facilitate the communication and the transfer of a meaning in the realm of mathematics.

The teacher will probably have to be familiar with the world of drama in order to better carry out his course sessions. Certainly, it will be interesting for the teacher to know the basic elements inherent in the development of a theatrical play and to practice them in a theatrical performance.

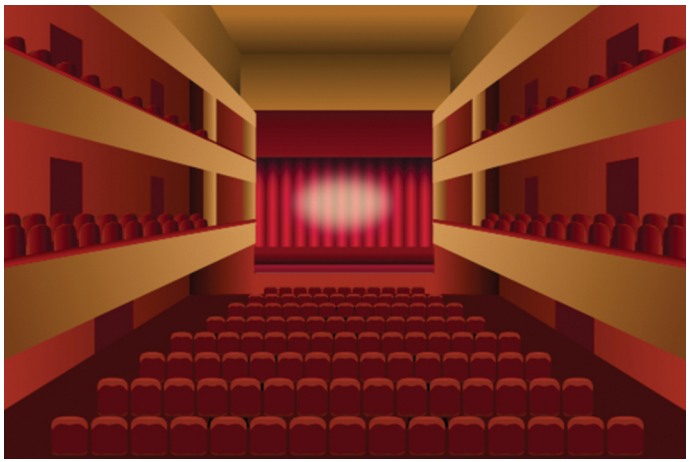
Becoming a teacher-director requires the ability to stage-manage and to design costumes and outfits that compliment the set and support the concept of the theatrical approach. The teacher should also be able to express the ideas he wants to transfer by writing or identifying a script related to the topic he wants to teach. For the latter, it is expected that he will be able to prepare in dialogue form or to identify from the existing literature or even to adapt some text so that it reflects the content and meaning of his lesson.

He has to know to a minimum extent of how scenography works like a real drama director. Some familiarity with the stage will be useful, although arrangements can be made depending on the imagination and creativity of each individual.

Let us observe an ancient and then a modern sketch of the stage, like Andre Degaine proposes. This can be proved interesting and useful for understanding the organisation of the stage. It also seems interesting to formalize the space and to put students in a real playing situation. It can be built in a semicircle, like those of antiquity:



Or in more modern way, all in depth:



"Théâtre à la Française"

And in fact, why not use those proposed arrangements in the classroom?

Obviously, you do not expect that in an ordinary classroom you are going to have these ideal spaces. The teacher will have to design and adapt the space that is available to him, so that it becomes as attractive as possible, aiming at creating the best possible background for his efforts and objectives. The following example is a simple approach of the stage in the classroom:

A theatre is easily created, the play can be played!

The layout of the scene is completely free. Moreover, we can see that the teacher is the facilitator and the implicitly in the centre of the discussions.

The role of the teacher director or any other facilitator for the development or performance of a theatrical play could gradually be passed to the students.

Drama undeniably leaves the teacher free to design the course and the show because, as Victor Hugo put it, in *"Faits et Croyances" (Facts and Beliefs)*: "A play is someone. It is a voice that speaks, it is a spirit that illuminates, and it is a conscience that warns."

The teacher-director will be delighted to discover his students under a new light, while teaching mathematics through acting.

To conclude, we shall simply quote a great lady: Ariane Mnouchkine:

"Drama is responsible for representing the movements of the soul, of the spirit, of the world, of history."

Section A5: Linking Theatre Scenario with Mathematics Curriculum

Assessing/Adapting/Writing/Preparing a Math-Theatre Presentation/ Scenario with Mathematical Context/Structure

PART 1

Drama in Education

An innovative way of learning is educational drama. It is different from other common forms of learning because the students extemporize playing a theatrical role. The drama is a powerful educational tool because of creating great enthusiasm and inspiration among students. This is the conclusion of several credible studies[1].

The art form of drama is used for all ages of students. It associates theatrical tips in order to improve the personality of all the students', to boost their social, physical and cognitive skills and to help their emotional evolution. It is a multidimensional way of knowledge planned to:

a. Raise the self-realization: mind and body and the cooperation and interaction with other people

b. Raise the clarity of expression and creativeness in communication

c. Help deepen the comprehension among human beings, the diversity of their views and their knowledge and appreciation of history and culture.

All the elements of theatre are used: such us scenery, lighting, props and costumes. Also, music and sound are used to enrich the game of learning. The education by drama comprises a wide range of subjects and kinds of theatre: drama games, mask theatre, clowning, pantomime, puppetry, improvisation, dramatic setting, and melodrama.

Drama in Mathematics

Mathematics is often faced as a dry, lonely subject without any kind of beauty. People think that the answer in a mathematical problem is right or false without

any exception. The mathematicians disagree with this opinion. They believe that the mathematics is an imaginative field of study and drama is a very helpful way to indicate this.

Nowadays, the student became from a "cognitive subject" a "social subject", meaning he is affected mainly by his culture and history. By a sociopolitical point of view, the question is how mathematics education is beneficial in personal life.

The offer of drama has been studied in many circumstances: such as in problem solving procedure, in several forms of teaching and in comprehension activities. Moreover, taking part in educational drama allows the students involved to have a joint experience. They engage in the learning process by undertaking a role that has to do with realistic problems. This way they are guided to think in a more realistic manner about the role, the problem in question and the interactions with their co-learners. As a result, they develop their critical thinking and problem solving skills, while at the same time they unravel their creative potential.

The international drama-in-education community[3] states that drama as a teaching tool can be used for all majors and across different school programs. Despite this, it is rather a novelty to use drama to teach mathematics. Studies show that the focus is given on the knowledge and comprehension of maths through the theatrical experience that is expected to to be beneficial for the students. In addition, it is widely known[2] that the use of drama for teaching mathematics increases significantly the students' understanding in comparison to textbook teaching. By working on improvisations in the classroom, students receive stimuli that boost their imagination, explore many different interpretations to a mathematical problem and enhance the further comprehension of a mathematical concept.

Research and examples of maths teaching through educational drama

I. The "Transformation" Project, UK 1999 – 2003 [4]

A "transformation" project took place in collaboration with the National Theatre and some primary schools in London in 1999. The selected schools were from the

area of the East End of London, a region synonymous to theatre. The main aim of the project was to upgrade the pupils' literacy and math skills as well as to build up their reliance.

It was an extended project that lasted more than three years. The approach that was followed was not the usual type of collaboration within schools and external arts organizations. In that case, the schools had a main role in deciding what the aims of the work should be.

The team worked with the same group of pupils each year, so it was possible to get an acceptable representative sample.

Each year, the broad outline for working with the children was similar, and it was divided into two periods. In the first one, a series of workshops took place inside the school. In the second period, five drama workshops were conducted.

A harmonious collaboration was established between students, class teachers and workshop leaders. The team of the project included: people with academic background in drama theatre with experience in "applied" drama, people that were trained performers and actors, with experience in running workshops, and persons who were professional storytellers and performance poets. The team also included writers, musicians, designers, dancers and other specialists.

The involvement of teachers varied from school to school, but as the project was being carried on they all gave an important feedback about the nature and the quality of the work. As the years went by, there were some variations in personnel composition and the workshop leaders. For example one school quitted the project after the first year, but then two other schools joined in. However the main core of schools remained constant over the three years of the project.

All the workshops were provided by taking into consideration the age range and different levels of the participants as far as drama, theatrical and educational experience were concerned. A workshop would typically include:

1. Welcome and short summary of previous meetings.

2. Warming up (e.g. activities that include making a circle and introducing yourself and also adding a funny piece of information about you that the others had to remember; like a city you've visited the previous year and begins with the same letter as your name).

3. Theatrical Games (e.g. the well-known "bomb and shield" game that helps participants to understand the space they have to use and has also been used by science communicators to give an example of the chaos theory to secondary school students! This game has everyone moving randomly around the available space and then each participant is asked to secretly choose two group members to be a "bomb" and a "shield". Consequently, they try to move in space avoiding the "bomb" or putting the "shield" between them and the "bomb". The result is a completely random movement around the available space).

4. Working in Pairs (e.g., one person is telling short tales like a typical journey to school. The partner then retells the narration by mimicry, using exaggeration and humor in the presentation).

5. "Automatic" writing, which includes picking words related to a particular theme and group them together to make sentences. Then use the favorite sentences to create an instant play that will be acted out during the session.

The impact indicators of the project ensured that the educational goals set by schools were achieved and also offered a broader experience concerning artistic, personal development and cultural aspects. During the project students paid for visits to the following professional performances: *The Ugly Duckling, My Fair Lady* and *South Pacific*. This was a new experience for many of the pupils.

During the three years of the project, a considerable increase was observed in the pupils' self-confidence, presentation skills, language and mathematics fluency and ability to participate in peer reviewing with their classmates. The age and the increase of maturity of the students during the three years of the project were also taken into consideration before reaching to any conclusions. Comparisons with control schools from similar home backgrounds reflected a highly significant difference in mathematics performance.

II. Drama and Teaching Maths – USA 2001[5]

The following text (in italics) is an article written by Professor Mark Wahl from the School of Education of John Hopkins University in Washington DC. In his text, professor Wahl describes in the most vivid and colorful way his own experience in using drama as an educational tool for maths teaching. It is presented here, because it includes many techniques and tips on making algebra and calculus – two topics that are difficult to visualize or dramatize – fun, by using imagination and theatre techniques.

My use of the "personal" side of numbers for instruction goes all the way back to when I was working on my master's thesis in maths. It required investigation of complex proofs in the University of Maryland library's musty maths journals. Commonly, while picking through a baffling formula for hours, I would gradually descend into that early sleep stage known as hypnagogic, where strange, dreamlike episodes often make people wake with a jerk.

In my dreams the mathematical entities I was studying would begin to animate, Alice-in-Wonderland style, becoming people with mathematical traits. That is, the negative numbers became negative; the powers did "power trips," and all were all trying to do complex "operations" on each other. There was some major episode going on with characters trying to resolve a problematic situation. While getting emotionally involved in this drama, I would pop awake; details rapidly fading, but with a fleeting sense of having observed a complex "soap opera."

Over my many years of maths mentoring students of all ages I have noticed this "opera" reappear in pieces as I search for metaphors and connections that convey maths concepts. For instance, when teaching addition and subtraction of negative integers, especially to preadolescents, I find that moods are the best metaphor. A -9 mood is pretty grumpy while a +20 is ecstatic.

The expression -7 - (-2) describes a person starting out with a -7 mood, receiving a compliment that removes (subtracts) -2 (two negatives) from his mood, and now he is in a -5 mood. Later, students can use the mnemonic shortcut that two dashes together, i.e., a - (-) can crisscross to form a +, making the expression become -7 + 2. Without conceptual development or the mood model, though, a student will retain no "gut feeling" as to why the answer to -7 - (-2) must be -5.

Continuing the personal approach, I speak of two different "lands," Multiplication Land and Addition Land. In multiplication land, there are factors that multiply, but there are other things that go on there like division, powers and square roots. In Addition Land, only addition and subtraction happen. Zero is the "nobody" of Addition Land because it goes over to, and adds with, a number and the number does not even think anything happened. It just shrugs and walks away unchanged. However, if zero takes a vacation and goes to Multiplication Land, look out! It feels very powerful as it annihilates anybody it comes into contact with! On the other hand, one is the "nobody" of Multiplication Land. When it goes to Addition Land, it can at least cause numbers gently to change.

The expression "5 to the zero power" means that there are zero factors called 5. This happens in Multiplication Land, so the absence of any factors, when nothing is happening, gives us the nobody of Multiplication Land, one. When "nothing is happening" there we must signify it by 1.

Most new learners think "5 to the zero power" should yield zero, the nobody of Addition Land, but "5 to the zero power" has no taint of Addition Land in it. (Of course, there are mathematical arguments for why "5 to the zero power" should be 1, but "dramatic" talk like this helps a learner to expect the correct concept.)

A last example (among many possible) of the use of personal references and drama to make number concepts meaningful and memorable is the simple teaching of addition facts. I like to speak of ten as the "big shot" or "ruler" or "king" of our number system. Some kids don't believe it is the most important number in Numberland. I say, "How do you find out who is, or has been, very important in a country? You look at the coins and stamps." If you are in Numberland you look carefully at the numbers. You'll find there's hardly a whole number that lacks the imprint of ten. There are numbers like six-teen (meaning six and ten) and six-ty meaning "six tens" and 6 (being one of exactly ten one-digit numbers) and one hundred (meaning ten tens).

Then how does 9 feel? (Almost important.) We could describe 9 as "Hungry for 1." So when it meets 7 it says "How would you like to hang around with a ten?" The 7 says "Wow! Of course!" The 9 says "You only have to make one sacrifice. You must

give up one and be a 6." The 7 says, "It is worth it!" and hands over 1, and together they are six-teen (six and ten). The moral of this story is that when 9 meets any number in Addition Land (even 47) it asks for one and becomes a ten.

The way I see it, the use of drama in communicating math is one way to tap the intrapersonal and interpersonal intelligences to teach mathematics.

III. Educational drama: A tool for promoting marketing learning?, Australia 2013[6]

Despite the fact that this example is about drama as a marketing educational tool, it is presented here for two reasons: firstly, this marketing class included many mathematical elements and the techniques presented are useful for teaching mathematics. Secondly, this is an example for using the educational drama for older students, which is still a controversial theme, as the people opposing this view claim that older students are not always happy to participate in drama activities. On the contrary, this study reflects a positive outcome out of the use of drama as an educational tool for a more mature target group.

Moreover, potential demographic differences in the perception of educational drama could be measured in a secondary level. It was concluded that educational drama is regarded as a highly effective form of learning by all participants.

Educational drama - Drama conventions

Drama conventions are methods used to run the process of educational drama. They are described as ways to interact imaginatively and to mix time, space and presence to serve the drama purposes while experimenting with different types of theatre. Conventions can be classified in four major groups:

1. Context building

Here the effort is focused in setting the scenery and adding info and context to unfold the drama, such as in sound tracking or defining space exercises.

2. Narrative action

Here it is all about the story, the action, the time, the change of the plot, etc. Exercise examples include meetings, or a-day-of-your-life type of narration.

3. Poetic action

This means the symbolic part of the drama, through intense use of carefully selected gestures and language such as forum theatre or mimicry.

4. Reflective action

This defines the inner thinking that forms the dramatic context with the most profound example being reflective narration or even head voices. In the ancient Greek plays, this role was performed by the "chorus".

The educational drama conventions methodology differs from traditional role-play for many reasons. It is focused in the process itself and not in the final exhibition; this means that participants use it to learn and not to demonstrate certain skills they have conquered. They are actively working on a variety of tasks such as researching, planning and presenting. The teacher or the instructor is not there to give prepared answers neither to tell participants what to do or what they will learn.

All students improvise, and there is no script available. This way, the same beginning may lead to different outcomes in different groups. Role building puts special emphasis on improvisation and students are encouraged to discover their own voice and personality.

However, the most important difference is the context. When using conventions, context is the most important element. What is said and done is shaped by the situations we get involved in and it helps in understanding the human behavior in various circumstances.

Traditional role-play usually works with practicing and rehearsing previously taught skills. In this case, students try to imagine what a different person would say or do in a specific situation, and they try to imitate his usual mannerisms such as his appearance and voice. Conversely, in drama they experience a specific situation as themselves.

Benefits expected from educational drama

In this study, thirty-two undergraduate students participated in various types of drama conventions. The advantages of this methodology included imagination and confidence boost, freedom of expression, application of ideas, critical thinking and deeper learning.

The disadvantages were mostly the long hours needed to dedicate on the project and the doubts on the suitability of the method for all the taught subjects. In general though, students found that the advantages outnumbered the disadvantages.

It was also found that the students experienced high motivational levels and a strong sense of realism. In addition, they highlighted that listening to different and unexpected opinions from people in different roles had an added value and made them more open-minded. Props, costumes and dramatic music also added to the experience.

Specified learning outcomes from the study

1. Raise awareness on major marketing issues and their impact
2. Build appreciation of the role of research in marketing
3. Develop skills as learners and practitioners
4. Conquer advanced communication skills
5. Practice in writing articles for popular marketing journals
6. Think and debate issues

Conclusions

The graphs bellow reflect the analysis of the outcome of this research. Students found the tested methodology (drama) to be a powerful educational tool, especially compared to more conventional tools like typical classes and lectures. It also helped them improve presenting, writing, understanding, working together and decision making skills.

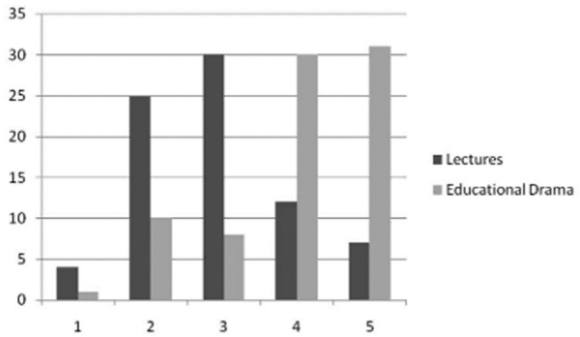


***Figure 1**: Respondents' views of how much they learn when educational drama or lectures is used as a learning method. Note: 1 = never learn anything, 5 = always learn a lot.*

Variable	Mean	Standard deviation
Communication skills		
"Doing educational drama has helped me develop my presentational skills" (Presentation)	3.6	0.643
"Doing educational drama has helped me develop my skill in writing" (Writing)	2.6	0.819
Learning		
"Educational drama helps me understand theoretical concepts" (Theory)	3.3	0.569
"Educational drama is helpful in understanding complex problems" (Understanding)	3.1	0.640
"I learn a lot when educational drama is used" (Learning method)	3.2	0.844
Social skills		
"Doing educational drama gives me the confidence to express opinions" (Confidence)	3.3	0.740
"Doing educational drama has helped me develop my team-working skills" (Team work)	3.6	0.644
Real world		
"Educational drama illustrates how business/marketing works in the real world" (Real)	3.5	0.577
"Educational drama helps me understand how business decisions are made" (Decisions)	3.2	0.612

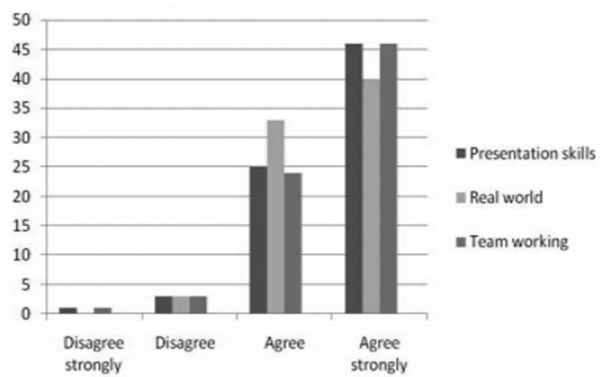


***Figure 2**: Respondents' perceptions of the nature of what they learn when educational drama is the learning method. Note: Statements used were "Doing educational drama has helped me develop my presentational skills", "Educational drama has helped me develop my presentational skills", "Educational drama illustrates how business/marketing decisions are made in the real world", "Doing educational drama has helped me develop my team-working skills".*

Applied drama techniques

On top of presenting a large range of characteristic studies and classroom examples, it is considered to be very useful for the educators to present a series of techniques associated with the applied drama.

Drama games

Drama and theatre games are introductory activities and exercises that are used for letting the students to understand what drama is all about. Activities like these tend not to be very intrusive and require high levels of participation.

Choral speaking

Choral dramatization asks students to read aloud and assigns parts to each participant. It uses texts such as poems or simple rhymes and illustrated books. Participants are able to experiment with different voices, sounds, gestures and movements.

Tableaux

Tableaux help students visualize pictures with their bodies, focusing on details and relationships. Tableaux are scenes frozen in time and usually involve at least three levels. Participants give emphasis on facial expressions and body language. This technique is helpful to develop the presentational as well as the communication skills of the participants.

Improvisation

Improvisation is dramatizing without a scenario and reacting in response to the environment stimulus. It can be a wonderful introduction to role playing. Students through their own position and expression improve their creativity skills.

Role playing

Role playing includes playing a character in a situation that might be real or imaginary in a variety of contexts. This technique can ideally be applied in many

subjects of the curriculum to support and strengthen comprehension of content. Below is a list of some common role-play strategies.

Reenactment

A historical setting or a specific story scene is required here. Despite the time period though, it is about "now" and things are happening in present tense. Students interact with written means of communication and develop characters based on that.

Extended role play

How is a scene continuing after its end? Or what brought things here? A prequel or a sequel of a specific event is used here based on the use of false cause and effect in logic.

Hotseat

Every participant is interviewed on his take on playing a character. This way further understanding of the role or the content is achieved. Other participants may also contribute by providing extra questions.

Expert panel

Students do their research and become experts. This way they understand what makes an expert and how deep is the field in question.

Writing in role

An alternative of the above strategies is to ask the students to produce written content while still playing a character. Thinking as a character in a specific situation will lead them to produce different pieces like a letter or a monologue.

PART 2

Adapting a more hands-on mindset: How to apply a theatre scenario in the classroom?

When the time comes for the educator to apply a good practice or a tested technique in the classroom, a need for more practical guidelines rises. This second part of the methodology aims to equip educators with practical information on how to apply the ideas mentioned in previous pages in their classroom.

Choosing your goal

The starting point of every educational action is to set goals. In this case both educational and theatrical goals need to be clarified.

From the educational point of view, the teacher needs to clarify what are the expectations he wants to achieve through this specific action. For instance, when Professor Theodore Andriopoulos wrote the script of the crime fiction play: "Who killed Mr. X?" he had a very specific goal in mind: he wanted his students to do a revision of the chapters he had covered during the school year. This is why the crime story he developed included mathematical quizzes based on exercises from each chapter of the textbook.

The structure of the story will be developed according to the educational purposes that will serve. For instance, if it is going to be a mathematics history narration, the structure will develop accordingly.

If it is going to be a problem solving skills development, the structure will be focused towards this goal and thus will be different than the previous one.

After setting the educational goals, the theatrical aspect should be taken into consideration too. The main question to be answered here is the following:

Is there going to be a performance?

Which is going to be the final outcome of the drama action? Is it going to climax and result in a performance or is it going to be used only as a classroom tool that will use the theatre convention to boost the comprehension of a certain topic?

On the one hand, working towards a definite product provides a concrete goal for the class and a source of motivation for the pupils. Care should be taken, however, not to detract from the importance of process during the classes. The emphasis should be given more on the preparation and the knowledge sharing than performance per se.

On the other hand, a performance requires a series of new elements and processes on its own; it might be difficult to present a complete theatre work, especially in a narrow time frame. A solution to this is to present a short show that would be about 10 minutes.

Yet, the educational drama is a technique that does not require a performance for the finale. Depending on the class and the messages that need to be conveyed, an educator could include theatrical exercises in their everyday teaching. For example, the educator could give roles to the students, like the role of the financial consultants of a large company that needs to cut down its operational costs by 20%. Students could debate on what costs to reduce by creating a budget and supporting it. This scenario will not result in a performance, but the students can still learn and comprehend deeper a topic, develop problem-solving, presentation and negotiation skills, and have fun at the same time.

Please note: from now on, the techniques analyzed will assume that the final goal of the maths drama will include a performance as a final result.

Working as a team

In order to achieve the maximum level of participation for all students, we should split them into teams. The teams should not be comprised by students of equal level of mathematics knowledge. Putting all the high performing students together would not work, as the other teams would be discouraged and not reach their maximum potential.

The number of the team members should vary between two and five people. Obviously, two people is the minimum to form a team, but a number larger than five could result in some students doing more work and some others to not participate as much.

Choosing your topic

The selection of the topic of the play, will be based on the educational purposes and needs of the class. Unless the educator already has a clear topic in mind, the students could easily be involved in this process. Moreover, giving them the chance to choose what they want to do, you can make them more engaged on the project.

A method to decide on a topic is the following: after discussing the educational goals and purposes with the students, give them some time to discuss and brainstorm on some proposed topics or let them suggest some topics individually or with their teams. You can ask each team to decide and propose a number of topics (usually three to five) to work on.

Afterwards, ask the teams to present and justify their ideas and write them down or on the board. Then, after each team has finished their presentation, summarize and list all the ideas and ask from the students to vote their top three choices. Depending on the atmosphere of the class, voting could be done by raising hands or by ballot.

The most popular topic will be decided by following this process and the entire classroom can start working on it collectively.

Sharing responsibilities

After the teacher splits up the classroom in teams, each team undertakes a certain duty. It is not effective for the students to do everything and odds are that not all students will feel comfortable being involved in every part of the process. For example some kids will not want to go on stage, while others will not enjoy to be involved with the writing. This is why each time they must discuss and decide what are their strong points and talents and consequently in which task they will contribute. This can be an alternative way of splitting students into groups as well. Decide early and all together on the various tasks like: script writing, acting, music

composing/ music selecting, direction and coordination, creation of costumes and props.

It is possible that after each team's tasks are decided, most of the students will have at least one specific preference. In this case, there is window for some flexibility to try to accommodate each student's wants, in order to ensure that everyone will participate in the team work.

Writing the script

The beginning

Everything begins when a basic outline of the script is ready. Students do not have to wait for the complete version to start working. For example, they can start creating costumes for the characters, build and study the characters personalities (especially if the characters are inspired by historic figures), compose the music and improvise.

Moreover, there is a common misconception that the students who will write the script are the ones who will comprehend deeper the mathematical topic in question. This is not accurate, as every student will work on the project from a different angle. For example, a person working on the props in a play based on mathematics in ancient Greece will learn that the mathematicians of that time did not use the ruler we now have, but they did everything using the gnomon. Plus, a good practice is to have the script team present its work in stages, in every meeting. In this way, all the kids get involved in mathematics and in the story development, while the script team develops its presentation skills.

But how do you actually start writing the script? The script is a complex process and it is going to be built through a series of different stages, but the starting point is always the most important. The teacher can help the students to take the first steps, by using some popular creative writing exercises. In this text two of them are going to be presented: the writing burst and the different point of view.

The writing burst

A writing burst is a 10 minute writing exercise. The teacher gives a topic and asks a group to start writing about it for 10 minutes, without worrying about the quality and the appearance of their work.

The thought of writing a short story by yourself can be frightening. It is much easier to set a timer for 10 minutes and begin writing without stopping or looking back.

How does this way of writing helps? This method is usually used by journalists or by writers, when they have very little time to write an article or when they want to create the mood for writing a bigger piece. Sometimes this material can be used as a starting point for a work. Moreover, writing bursts help people to generate innovative ideas, because they write without stopping or looking back and they do not have to worry about spelling or grammar mistakes.

The different point of view

It is funny to think about narrating the "Three Little Pigs" story as the big bad wolf. Write a title like "the true story" and start working on presenting the story from this point of view. Or how about writing the true story of "Cinderella" from the point of view of the two bad step-sisters?

And now let us imagine how these can be applied in maths. For example imagine the birth of number zero from the point of view of other numbers. All the other numbers think it has no value at all, until it pairs with one of them… Moreover, spare some moments of thought on the Pythagoreans… besides of the famous theorem, research the true story of the Pythagoreans, this strict community. Will a rejected student stay alive to tell his story? Or imagine, as happens in Flatland, a rectangular telling the most unlikely story of his three-dimentional adventure, in prison, alone and disconsolate because nobody believes it.

Let the children consider what they know about the mathematics topic of their interest and let them imagine and write another version from another point of view.

An inquiry about the topic should follow and the results could be announced in the classroom. This procedure could bring new innovate ideas on the table and inspire students.

Script building

After gathering all the necessary information, building the story comes next. The five "W"s are the key to unlock any difficulties you might encounter in putting your ideas in order: where, when, what, who and why?

Where and when did the play happen?
The answers here could vary from historically accurate (in the library of Alexandria at 200 BC) to fully imaginative (in a planet hundreds of light years away).

What happened (exactly)?
The facts should be put here in order to unravel the story.

Who did all these?
Is the main character going to be a historical person? Is it going to be an imaginary one? Or is it going to be a personalized math symbol or idea? For example a function that is depressed because it has its concave down and decreasing.

Why did this happen?
The aftermath and the moral of the play can be found by asking why. What were the character's motives to act like this? Did the general situation accelerate things and set the plot into motion? What about the politics or the social factors of the time?

How did this happen?
This is a bonus question, giving us space to further develop and flesh out the story. It is the question that asks about details and ideas and the question that takes the writer deep into the heart of the action.?

Also, a script analysis would never be complete if the three backbones of a play, originating from ancient Greek drama and Aristotle's theatre analysis were not mentioned: **Mythos** – **Ethos** – **Setting**.

Mythos is all about the story and how to create a narrative that could captivate the audience. The story needs to be good, to have a climax and plot twists. A usual structure is like this: the characters are introduced and their normal, everyday lives are presented. Then something happens unexpectedly that disturbs their daily routine; the characters decide to do something about it or something happens and sets everything in motion. The characters set a goal, but they have to face difficulties and to struggle in order to make it.

Depending on the message and the feeling of the story, the characters succeed or fail in their quests. In the end a new status quo, a new "the way things are" is reached which is different than "the way things used to be" and this is why the story is important and significant: because it changed things.

Ethos is all about the characters. Who are they? What is their story and what are their motives? Strong character building includes adding some core characteristics that will guide the character's actions. To understand what that means, remember Ebenezer Scrooge. He was such a powerful character that calling someone Scrooge today is a sign of this person's meanness, misery and lack of generosity. He managed to escape the pages of the Christmas Carol of Dickens to find a place in our everyday vocabulary. Similarly, Heidi brings in mind a young girl running in the mountains. This indicates that a character can come to our minds because of particular elements of his appearance, distinct traits of his personality or because of his actions.

Setting is what the word indicates: everything else that form the environment of the story and its atmosphere, everything that answers the first two "**W**"s, the where and when. This is about the place, the time, the scenery and the feeling.

After having created the story, the next step is to trim it down. Keeping it short is one of the little success tips that do not come easily, as no one is happy to see the text he/she wrote to be cut. However, this step is necessary and the best way to correctly identify what should stay and what should go is to read loudly the text to an audience (in this case the rest of the class). There will be spots where even the narrator will want to pass on more quickly.

The devised theatre approach

A different approach for the script writing and development is the devised theatre approach. In this case it is not the team of the writers that leads the story writing, but the team of the actors who improvise on given facts and create on the spot and from scratch dialogues, maneuvers, attitudes, behaviors and in the end, characters.

Depending on the students, this relatively new theatre technique could work in the classroom, provided that students will not be afraid to try and build something

from scratch and that they will be committed and serious enough to create the characters and the scenes through collaboration and team work. In this case, the writing team will decide from where the improvisation will begin and will write down all the dialogues and scenes that will be tested before deciding the final text.

And after writing the script, what?

The question that rises here is simple: in a team project that includes rehearsals and scene preparations, what is going to be the role of the writing team after the script is written?

There are two paths to follow here (the one does not exclude the other). If kids wish to do so, they could split and join other teams and keep on in a different field. But if kids do not wish to get involved in different activities, they could be very useful during rehearsals and preparation by being quality evaluators. This new role includes making sure their work is understood and consulting the other students on how to interpret on stage the script.

Rehearsals and Preparations

With the script ready, it is time to go on with the rehearsals and to set the stage for the performance. A whole new world of tools and techniques takes part here and it is presented in the chapter about The Theatrical Approach of this guide. This part includes the music, the coordination, the fine tuning, the costumes, the props and the settings.

The final question we will try to answer in this chapter is the following: creating a play, let alone a mathematics play, is a time consuming work. Does it fit inside the school curriculum or would it better fit as an extra-curricular activity? In the majority of times this is something to be decided among the teacher and the school but the usual path is to do both: start working during the regular courses of mathematics, then maybe collaborate with the theatre teachers and use some of their time too if it is possible. You can add extra hours as the final show will come to an end.

The performance

When the day before the performance arrives, students are usually (over) excited and educators have to increase their fine tuning and coordinating efforts. A performance is something that is both fluid and dynamic and it usually brings good cheer to both participants and viewers. It is important for the students to have fun during the play, and negative feelings caused by anxiety or perfectionism should be cast away.

A performance is like a test; there is no point in doing last minute studying/ corrections. What has not been rehearsed many times will most probably be forgotten. The students should be aware of things that could go wrong and should plan accordingly on how to improvise for covering them up. For example, if there is a costume fault and the moustache of someone is ready to come off, then he should continue and not freeze or lose his words. The best thing to do is to prepare some easy lines that will comment on the moustache's free fall to provoke the audience's laughter.

When something goes bad on the scene, professional actors usually comment on it in a humorous way. In this way the audience gets more engaged. The other solution is to ignore it. If something is lost or missed, improvisation is again in order – keep in mind that the audience has no idea of how the play evolves; consequently, there is no right and wrong as far as the show goes on without pausing.

Assessing the project

When is a maths theatre project considered to be successful? In general terms, a MATHeatre scenario is successful if it fulfills the educational goals set by the class, engages the students to work creatively and collaboratively and gives them a fresh look on maths learning while having fun. Some predefined criteria could help, like the accuracy of the content, the message effectiveness, the involvement of the students, and the amount of creativity and imagination involved. Moreover, feedback from the audience and peer reviewing are useful tools to receive an objective evaluation.

Example:

Assessment criteria of the theatrical activity in class

The two empty columns are here according to different ways of marking in the different countries in Europe (A,B,C/0-10/no mark).

I. Mathematical content

The student has approached a concept studied in class		
The student was able to stage clearly the concept		
The student has represented a theoretical concept within a support		

II. Theatrical aspect

The student feels at ease/confident in front of the other and expresses himself correctly		
The student uses well the space		
The student has respected the given instructions		

III. Creativity of the staging

The student is involved in his/her delivery and gives consideration to the other actors		
The student demonstrates originality (e.g., music)		

Adapting a scenario

In some cases, when time is limited, or if there is a nice play that is inspiring the students or the educator, they could consider adapting a scenario. This could also be the case in adapting a book or movie to a play.

The first thing you must think before any adaptation is the intellectual property rights. Usually every author of the original text material holds the copyright. This means they have the right to say if a play can or cannot be made based on their material. If the answer is yes, how much will it cost?

The legal and correct thing to do is to check and get contact with the writer, so you can start the procedure of buying or optioning the rights. Sometimes if the material is used and adapted for educational purpose, dispensation is free.

Additionally, because of the copyright expiration, if you are interested to adapt a text written in the 18th century, the work is considered to be "in the public domain" and you have not the obligation to secure any rights at all.

After clearing out the copyrights, the question is how to adapt the story. The methodology of work is the same with the one needed to write a story. Unless already existing, dialogues should be written or localized or adapted to the class's special needs. This means a writing team should lead this work, the same way it would lead the work of creating an original script. The team can still do some research on the facts and details of the original material, present it to class, decide what is going to stay and what is going to be excluded in the final script and then write it down. Even if the class works with a devised theatre technique, its collaborative improvisations may be based on the original material of a book or a movie or an already existing play.

What should be kept in mind is that each team has its own character, and this means that the material that will be adapted will be transformed, according to the team's needs and dynamic. Teachers and classes may choose to stay close to the spirit of the material or to use it as a starting point. In any case, they should discuss a priori why they chose it in the first place. They should discuss the messages that attracted them in the story and to make sure they will keep them and present them in their final production.

Section A6: Improving Mathematical Competences

Modern technologies have a major impact on the world around us. They significantly affect the ways we communicate, think and access information. Change happens at an increasing pace and new breakthroughs or technological advancements do not take decades or centuries any longer, but years. This can be illustrated on the example of web services that have heavily influenced people's lives – Wikipedia, Google or Facebook – in the last few years. Of course, this rapidly changing world puts much pressure on school, which is meant to prepare pupils for later in life. It is no longer sufficient to just transmit knowledge or procedures. Schools must develop highly transferable and adaptable abilities (key competencies) that pupils will be able to apply in their daily lives, after graduating from school.

Key competencies represent the system of knowledge, skills, abilities, attitudes and values that are important to the individual's personal development and to the individual's role in society. The selection and the concept of key competencies are based on values that are accepted by society and on shared ideas, such as the competencies of the individual that contribute to his or her education, welfare and success in life and to the strengthening of the functions of civil society.

Key competencies are not isolated phenomena; they are mutually linked and intertwined, multifunctional, have an interdisciplinary nature and can only be acquired as a result, of a comprehensive education process. Therefore, their forming, shaping and development must be the ultimate aim of the entire educational content and of all the activities taking place at school (Framework Educational Programme for Basic Education, Czech Republic).

Key competencies influence teaching in all subjects across the curriculum, including mathematics. The goals of mathematics education are partly with respect to the development of these key competencies. However, besides the development of key competencies, mathematics education has its own objectives based on mathematical content.

Mathematical competence is the ability to develop and apply mathematical thinking in order to solve a range of problems in everyday situations. Building on

a sound mastery of numeracy, the emphasis is on process and activity, as well as knowledge. Mathematical competence involves, to different degrees, the ability and willingness to use mathematical modes of thought (logical and spatial thinking) and presentation (formulas, models, constructs, graphs, charts). [Recommendation of the European Parliament and of the Council of 18 December 2006 on Key Competences for Lifelong Learning (2006/962/EC)].

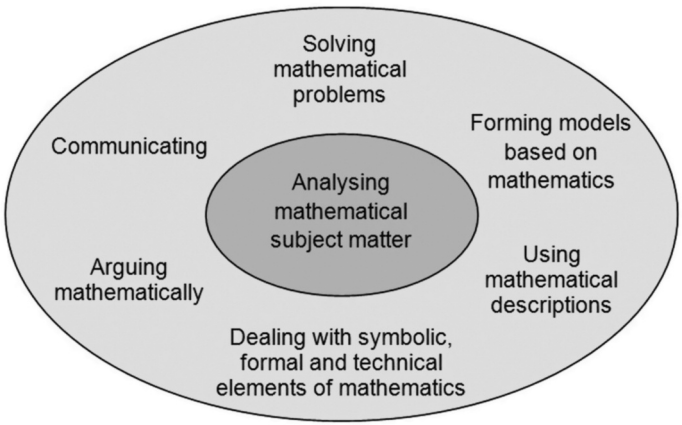


Figure 1 - General competencies [www.sinus-transfer.eu]

Mathematical competence is necessarily connected with mathematical knowledge and skills and cannot be discussed separately. Elementary knowledge of mathematics is crucial for the development of competencies. This prerequisite fundamental knowledge of mathematics includes a sound knowledge of numbers, measures and structures, basic operations and basic mathematical presentations, an understanding of mathematical terms and concepts, and an awareness of the questions to which mathematics can offer answers.

An individual should be able to reason mathematically, understand mathematical proof, communicate in mathematical language, and to use appropriate aids.

A positive attitude in mathematics is based on the respect of truth and willingness to look for reasons and to assess their validity. [Recommendation of the European

Parliament and of the Council of 18 December 2006 on Key Competences for Lifelong Learning (2006/962/EC)].

Theatre and theatre activities in mathematics lesson can contribute to development of both key and specific mathematical competencies. The following text focuses specifically on their benefit for mathematics.[1] Based on analysis of examples of good practices in countries around the world (see Good practices Report Math Theatre, www.le-math.eu), we identified the following areas in which the use of theatre activities contributes to development of mathematical competences:

1. Dramatization and mathematization of a situation

Contemporary educational strategy of mathematics in the majority of classes is topic oriented. Dramatization helps to develop the ability and willingness to use mathematical modes of thought. An example of such activity is the activity Autobus, in which mathematical problems are modelled in the context of bus transport.[2] Similarly children can act out other situations in which they e.g. go shopping, order in a restaurant and many other situations in which calculations are carried out within the context of a real-world situation. The task assignments are often modified by the actors themselves as they react to how the situation develops. Thus they learn to perceive and understand the complexity of the real world; gain experience in the use of mathematical modelling (apply mathematics to practical situations).

Theatre forms, within the framework of dramatization, also help to develop the ability to co-operate while solving problems, which reflect real life situations and promote learning regarding the possibilities of mathematics in real life.

1. *The general benefit of theatre activities is discussed in detail within the frame of the subject Drama education.*

2. *The bus route is marked by several (for example five) stops in the classroom, which are labelled A, B, C, D, and E. The stops are at particular places in the classroom, e.g. the teacher's desk, a washbasin, map, whiteboard, wardrobe, the piano ... On each stop pupils act out getting on and getting off the bus [Hejný, 2008, on-line: http://www.cme.rzeszow.pl/pdf/part_1.pdf#page=40].*

Figure 2 - Example of shopping from ZS and MS Pisecna

2. Dramatization and visualization of a mathematical situation

Dramatization may also be used for illustration of situations of purely abstract nature. An example of such activity is visualization of solution of linear problems, in which pupils act out their own solving procedures (see figure 3).

Theatre forms in dramatizations develop processes, such as analyzing problems and planning solutions, choosing the proper approach to resolving a problem, evaluating results for correctness with a view towards the nature of the task or problem.

3. Theatre and history of mathematics

Theatre can also be used to introduce pupils to the history of mathematics and historical contexts, in which different discoveries were made. These activities help to develop not only cross-curricular thinking, but also to develop abstract and precise thinking by acquiring and using basic mathematical concepts and relationships. This can be achieved by recognizing their characteristic qualities and identifying and classifying concepts on the basis of these qualities.

Figure 3 - Solving.equations
http://www.dailymotion.com/video/x6p7h8_mathematique_creation#.UcFkydgriZc

Figure 4 - Activity from Math Theatre 2010

4. Improvisations in mathematical context

The activities which require improvisation are also very important for the development of mathematical competence, as long as they require pupils to use mathematical concepts. An example of these activities may be the task to explain a mathematical concept, using pantomime or verbal description, without using words with the same stem. Pupils thus learn to express themselves precisely and laconically by using the language of mathematics, including mathematical symbols and they are also able to distinguish between substantial and insignificant properties of a given object.

Section A7: Motivation and MATHeatre

In the contemporary secondary school, motivation is a topic that occupies a central role. Motives create the main conditions for promoting activity and achievement. Without the necessary motivation, it is not possible to successfully realize any type of activity and we are in danger of producing inadequate results. The way students feel in given situations affects the volume of the efforts they apply when studying. That is why it is important for the whole educational process to generate passion for knowledge and tense mind labor. It is quite logic to conclude that the success of students is only possible when learning becomes a desired process. It can be argued that the learning process is comprised by a system of motives, which includes: cognitive needs, goals, interests, aspirations and ideals. The above are motivational elements with an active and purposeful character, which can help us to determine the content-meaningful peculiarities of the individual. This specified system of motives helps us to organize learning activities and is characterized by its stability and dynamism. Learning motives determine the outcome and the direction of the learning process and help us to hierarchize the desired educational results. Considering the above, it can be argued that the motivation for studying could be defined as the sum of internal and external stimulations that helps students to trigger their desire for learning and fuels their passion for knowledge.

Motivation can serve several purposes: it can inspire, direct and organize positive behavior and it attaches meaning and significance to actions. Ideally, each

educational activity should address the needs of the student. In this context, it can be argued that a need is a mental state, which can potentially create the right conditions for a course of action. In the absence of those needs, the student remains passive and without a clear set if goals. It is also far more difficult to be motivated and to learn. In the educational process motives often relate to mastering knowledge, acquiring good grades and parent and teacher praising. Usually, motivation is preceded by the formulation of numerous objectives. Following this, students should be able to understand the purpose of an activity and when possible to propose, according to their opinion or interests, other desired and meaningful activities. When students are able to understand the meaning of an activity or an action it is far easier to be interested or engaged in the learning process. Interest plays a very important role in the educational activity and it is a principal component of the motivation process and it can often be characterized by emotional variety. The interplay between positive emotions and interest is very important during the first steps of engagement and active participation with a subject.

It is possible to use learning methods in the classroom, which enable the development of the aforementioned conditions, including skills for the expression of thoughts, skills for clear and exact knowledge structuring and skills for the establishment of solid contact and communication between the trainers and the trainees. The achievement of such results in the educational process requires the application of an interactive technique and co-ordination with the more traditional methods. A possible technique is the theatre. The main characteristics of the theatric educational environment are related to the following: augmented student activity and teamwork; change of the traditional roles of trainers and trainees; interactive method of work; effective organization of time and space; combination of various forms for interaction based control.

In a theatre based education the teacher assumes a supportive role and he ensures the effective organization of the educational environment. Additionally, he gives advices and instructions; he processes feedback, takes care of modeling and analyzing and proposes solutions to problems. The students are included in the process, and each of them contributes individually in the exchange of knowledge, ideas, means and actions. All these are created under an educational process in an atmosphere of well-intention, emotional and intellectual comfort.

Theatre based education is based on dialogue. It produces interaction between trainees, helps them gain mutual understanding and promotes team based problem solving abilities. In MATHeatre interaction can be distinguished in two different types:

- **Instructive model**: students study by acting in a theatre play, which has been created by somebody else or watch a theatre play whose actors are their classmates.

- **Constructive model**: students study by acting in a theatre play, which has been created by themselves.

Studying during the theatre performing process can incite an array of emotions. Theatre leaves an inspiring environment, which is related to two domains: the emotional one – personal (interior) motivation, emulation, curiosity, confidence and the cognitive one – significant and relevant context, active participation in “dialogue story," organization, different situations, feedback, support of students in knowledge organization.

Practically, MATHeatre is a simulation that requires the active application of the mastered knowledge. The participants perform definite roles or are an active audience. The effectiveness is rather high compared to traditional practices. The theatre plays are amusing and attractive for the participants. They encourage and enable communication, increase the learning interest and develop the independence of students. The playing activity’s educational purposes are based on the principles of activity, dynamism, entertainment, role performance, team character (team work), action modeling, feedback, problem collectivity, competitiveness, effectiveness and system.

The participants can combine fantasy and reality in their consciousness with respect to a function or activity and, as a result, they can study by acting. They simulate a defined situation in order to play what they have learned, or they master it in order to develop new abilities. In this way the participants elaborate various social skills: for communication – formulation of positions, listening to opinions, verbal and non-verbal expressing; for collaboration; for contracting; for avoiding or overcoming of conflicts. In the base of the theatre play, an understanding of

social roles is being discovered. These social skills can be considered the behavior forms that the individuals examine and build their social life upon. As a result, students socialize and they become familiar with the behavior forms which are expected from them.

The learning by theatre process places great importance on the team goal and the team success, which are achieved as results of independent individual work and by the continuous interaction between team members when working on the posed topic. The whole team (group) is interested in a positive final result and its members do not compete with each other. The main principles of this process are:

- one problem for the whole group
- one award or evaluation of the whole group
- distribution of roles under equal possibilities

MATHeatre gives chances for the optimal solving of a variety of didactic problems, which could be divided into three large groups:

- theoretical (motivation for theoretical preparation, formation of a proper system for comprehension, competences and concrete ways for their practical use).
- experimental (with possibility to check complex preparation).
- expert (the trainees could take different positions – as acting persons, who look for correct solutions or as experts, who analyze and evaluate the undertaken solutions).

The situational didactic plays could be divided into five basic features:

1. According to the character of the situation: reality; fantasy; rivalry; discussion; training.

2. According to the character of the theatre play: group opposing (interaction among groups); competition.

3. According to the way of presentation and the process of information: theatre with the teacher in a leading role; theatre with computer aided technology.

4. According to the dynamics of the modeled processes: theatre with a limited number of steps (limited time); theatre with an unlimited number of steps (unlimited time); self-developing theatre.

5. According to the level of complexity: complex theatre staging (multilateral group interaction and a large number of connections); theatre staging with medium complexity (medium number of connections); non-complex theatre staging with a limited number of connections and without group interaction.

In MATHeatre's case, all five realizations of theatre's didactic functions are possible. MATHeatre is a typical example of active learning and interaction. In it, the process of acquiring new knowledge and skill mastering is achieved by including the trainees into various educational activities, related to theatre. The traditional atmosphere in the classroom (short or broken classes in lecturing style, in which the teacher is a central figure, while the students remain passive) is replaced by carefully prepared educational activities, in which the students have a central role.

MATHeatre participants examine their actions and experiences and try to continually improve. This method of learning is in contrast with the more traditional approach, which is concentrated on knowledge and skill presentation. The focus of MATHeatre is to investigate the executed actions and the acquired knowledge of the student in order to improve skills and academic achievement. Learning requires well-structured programmed knowledge and good planning of the questions and material that is going to be used, during the preparation stage of the theatre play.

As a result, the effectiveness of the educational process is increased. Furthermore, it should be highlighted that the knowledge acquired with this method is characterized by higher durability and profoundness. By participating in theatre plays students develop personal qualities like active participation, initiative, speed and new socially significant tools for behavior and communication. The nature of the activities is characterized by a rich and varied environment that promotes positive thinking, offers room to the students to express themselves and helps them to transform information into personal knowledge.

Section A8: Communication Skills and MATHeatre

Communication is a complex way of transferring information (content, message signal) between two parts, the sender and the receiver, using a combination of methods (written words, nonverbal gestures, spoken words). We use it also to establish and modify relationships. In some cases, communication is considered only the verbal communication, and the other, non-verbal communication aspects are regarded as part of metacommunication, which can influence the affectivity of communication. We will use the terms of verbal and non-verbal communication.

Elements of successful communication: MATHeatre show will be introduced.

1. Understanding your audience
2. Preparing your content
3. Delivering confidently
4. Controlling the environment

Understanding your audience

The success of presentations is mostly judged by the audience's responds. Before you even begin putting your PowerPoint slides together, the first thing you need to do is to establish your audience's needs. Try following these steps:

Determine who are the members of your audience and what kind of background knowledge they possess. Find out what they want and what they expect from your presentation.

What do they need to learn? Do they have any special interests that you need to respect?

Create an outline for your presentation, and ask in advance for feedback on your proposed content.

If you satisfy your audience's expectations (you see nods and smiles, or hear murmurs of agreement), it does not matter if your delivery was not perfect. The

primary aim of the people listening to your presentation is to get the information they need. When that happens, you have completed your task successfully.

Preparing your content

The only way to fulfill your audience's demands is to deliver the content they want: understanding what to present, and how to do it. If you provide the information in a well-structured format, and if you include different techniques to keep the audience interested, then they will probably remember what you said – and they will remember you.

There are a variety of ways to structure your content, depending on the type of presentation you'll give. Here are some principles that you can use:

Identify a few key points – To help the audience understand the messages you are giving them, use the chunking principle to organize your information into five to seven key points.

Do not include every detail – Good presentations inspire the audience to learn more, and ask further statements to maximize their understanding of the topic.

Use an outline – At the beginning, tell your audience what you intend to cover, and let them know what to expect. This raises their interest from the beginning.

Start and end strongly – Capture people's interest from the very first moment, and leave them with a message they will remember. Do not put all of your efforts into the main body of the presentation. If you fail to get your audience's attention at the beginning, they will not concentrate.

Use examples – use lots of different examples to support your ideas: stories, real-life examples, metaphors to make their mind busy.

A special type of presentation is one that seeks to persuade. Monroe's Motivated Sequence, consisting of five steps, offers you a framework for this type of presentation:

Get the attention of your audience – Use an exciting 'hook' or opening point, like a shocking statistic or a motivating picture. Be provocative and stimulating.

Create a need – Convince the audience there's a problem, explain how it affects them – and persuade them that things need to change.

Define your solution – Explain what you think needs to be done.

Describe a detailed picture of success (or failure) – Give the audience a clear vision; something they can see, hear, taste, and touch.

Ask the audience to do something urgently – Get the audience involved right from the start. Keep them busy.

In connection with persuasion, look at The Rhetorical Triangle. Consider your communication from three perspectives: those of the writer, the audience, and the context. It is a method that builds credibility, and ensures that your arguments are logical and easy to follow.

These tips may help you:

Practice to build confidence – if you practice, your speech will sound natural and genuine. Don't necessarily memorize your presentation, but be so familiar with the content that you are able to speak fluently and comfortably, and adjust as necessary.

Be flexible – This is possible only if you know the material. Never present anything you just learned the previous night. If you are not sure about something, just admit it, and try to find the answer.

Welcome statements from the audience – This is a sign that a presenter knows his/her topic. It builds audience confidence, and people are much more likely to respect your knowledge.

Use visual aids – Know the exact amount of visual information so as not distract your audience from what you are saying.

Keep your visuals simple and brief – Too many pictures, charts, or graphs are unnecessary. Your slides should only draw attention to the main message. Never burden your audience with every single detail. Slides should only concentrate on the overall message.

Manage your stress – Confidence is in connection with managing your stress levels.

If you feel too nervous before a presentation, try some of these stress management tools:

- Use physical relaxation techniques, for example, deep breathing and visualization, to calm your body and ease your tension
- Imagine yourself delivering a successful presentation while keeping calm
- Learn strategies to build your self-confidence in general. The more confident you are about yourself and your abilities, the more natural you will feel in front of people.

When you present with confidence and authority, your audience will probably pay attention to you as someone who is worth listening to. So 'pretend' if you need to, by turning your anxiety into creative and enthusiastic energy.

Delivering confidently

Pay attention to body language.

Types of Nonverbal Communication

According to experts, most of our communication is nonverbal. Every day, we respond to thousands on nonverbal cues and behaviors including postures, facial expression, eye gaze, gestures, and tone of voice. From our handshakes to our hairstyles, nonverbal details reveal who we are and impact how we relate to the others.

Scientific research on nonverbal communication and behavior began with the 1872 publication of Charles Darwin's, *The Expression of the Emotions in Man and Animals*.

The Major Nonverbal Communication factors

1. Facial Expression

Facial expressions are responsible for a huge proportion of nonverbal communication. Consider how much information can be conveyed with a smile or a frown. While nonverbal communication and behavior can vary dramatically between cultures, the facial expressions for happiness, sadness, anger and fear are similar throughout the world. Think for a moment about how much a person can convey with just a facial expression. A smile can indicate approval or happiness while a frown can signal disapproval or unhappiness. In some cases, our facial expressions may reveal our true feelings about a particular situation. While you may say that you are feeling fine, the look on your face may tell people otherwise.

Emotions expressed through facial expression include happiness, sadness, anger, surprise, disgust, fear, confusion, excitement, desire. Researcher Paul Ekman has found support for the universality of a variety of facial expressions tied to particular emotions including joy, anger, fear, surprise and sadness.

2. Gestures

Common gestures include waving, pointing, and using fingers to indicate numeric amounts. Other gestures are arbitrary and related to culture.

3. Paralinguistics

Paralinguistics refers to vocal communication that is separate from actual language. This includes factors such as tone of voice, loudness, inflection and pitch. Consider the powerful effect that tone of voice can have on the meaning of a sentence. When said in a strong tone of voice, listeners might interpret approval and enthusiasm. The same words said in a hesitant tone of voice might convey disapproval and a lack of interest.

4. Body Language and Posture

Posture and movement can also convey a great deal on information. Research on body language has grown significantly since the 1970's, but popular media have focused on the over-interpretation of defensive postures, arm-crossing, and leg-crossing, especially after the publication of Julius Fast's book Body Language. While

these nonverbal behaviors can indicate feelings and attitudes, research suggests that body language is far more subtle and less definitive than previously believed.

According to various researchers, body language is thought to account for between 50 to 70 percent of all communication. Understanding body language is important, but it is also essential to remember to note other cues such as context and to look at signals as a group rather than focusing on a single action.

5. Proxemics

"Personal space," is also an important type of nonverbal communication. The amount of distance we need and the amount of space we perceive as belonging to us is influenced by social norms, situational factors, personality characteristics and level of familiarity. For example, the amount of personal space needed when having a casual conversation with another person usually varies between 18 inches to four feet. On the other hand, the personal distance needed when speaking to a crowd of people is around 10 to 12 feet.

6. Eye Gaze

Looking, staring and blinking can also be important nonverbal behaviors. When people encounter people or things that they like, the rate of blinking increases and pupils dilate. Looking at another person can indicate a range of emotions, including hostility, interest and attraction.

7. Haptics

Communicating through touch is another important nonverbal behavior. There has been a substantial amount of research on the importance of touch in infancy and early childhood. Touch can be used to communicate affection, familiarity, sympathy and other emotions.

8. Appearance

Our choice of colour, clothing, hairstyles, accessories and other factors affecting appearance are part of nonverbal communication. Different colours can evoke different moods. Appearance can also alter physiological reactions, judgements

and interpretations. First impressions are important, not only when falling in love but also while public speaking.

Stand up straight, take deep breaths, look people in the eye, and smile. Don't lean on one leg or use gestures that feel unnatural.

Many people prefer to speak behind a podium when giving presentations. While podiums can be useful for holding notes, they put a barrier between you and the audience. Instead of standing behind a podium, walk around and use gestures to engage the audience. This movement and energy will also come through in your voice, making it more active and passionate. Pay attention to your gestures. Do they appear natural? Make sure that people can see them.

Last, look at how you handled interruptions, such as a sneeze or a question that you were not prepared for. Does your face show surprise, hesitation, or annoyance? If so, practice managing interruptions like these smoothly, so that you are even better next time.

More useful hints

Think Positively

Positive thinking can make a huge difference to the success of your communication because it helps you feel more confident.

Visualize giving a successful presentation, and imagine how you'll feel once it is over and when you've made a positive difference for others. Use positive affirmations such as "I am grateful I have the opportunity to help my audience" or "I am going to do well!"

Cope with Nerves

Many people cite public speaking as their biggest fear, and the fear of failure is often at the root of this. Public speaking can lead your "fight or flight" response to kick in: adrenaline courses through your bloodstream, your heart rate increases, you sweat, and your breath becomes fast and shallow. Although these symptoms can be annoying or even debilitating, the Inverted-U Model shows that a certain

amount of pressure enhances performance. By changing your mindset, you can use nervous energy to your advantage.

First, make an effort to stop thinking about yourself, your nervousness, and your fear. Instead, focus on your audience: what you are saying is "about them." Remember that you are trying to help or educate them in some way, and your message is more important than your fear. Concentrate on the audience's needs instead of your own.

Use deep breathing exercises to slow your heart rate and give your body the oxygen it needs to perform. This is especially important right before you speak. Take deep breaths from your belly, hold each one for several seconds, and let it out slowly.

Crowds are more intimidating than individuals, so think of your speech as a conversation that you are having with one person. Focus on one friendly face at a time, and talk to that person as if he or she is the only one in the room.

Watch Recordings of your Speeches

Whenever possible, record your presentations and speeches. You can improve your speaking skills dramatically by watching yourself later, and then working on improving in areas that didn't go well.

Controlling the Environment

- Try to reduce potential risks to your presentation
- Practice in the presentation room – Become familiar with the room and the equipment
- Do you have trouble accessing your PowerPoint file?
- Does the microphone reach the places you want to walk?
- Can you move the podium?
- Are there stairs that might cause you to trip?
- Do your own setup – Don't leave this to other people

- Test your timing – Calculate how long each part of the presentation will take, and this helps you plan how much time you'll have for statements and other audience interactions
- End your presentation on time. Be considerate, and stick to your agenda as closely as possible
- Key Points
- Presenting is scary but unavoidable. Find opportunities to practice the tips above.

Strategies for Becoming a Better Speaker

Public speaking is a learnable skill. So as to become a better speaker and presenter:

- Plan Appropriately
- First, make sure that you plan your communication appropriately. Think about how you will structure your message
- Think about how important a book's first paragraph is; if it does not grab you, you are likely going to put it down
- Start with an interesting statistic, headline, fact, or story as a powerful opener
- Expert Interviews with Annette Simmons and Paul Smith offer some useful tips on doing this.

Planning also helps you to think on your feet. This is essential for unpredictable question and answer sessions or last-minute communications.

Tips

Remember that not all public speaking will be scheduled. You can make good impromptu speeches by having ideas and mini-speeches pre-prepared. It also helps to have a good, thorough understanding of what's going on in your organization.

Practice

As the proverb says, "Practice makes perfect!". To practise, try to find possibilities to speak in front of others (for example making toasts, cross-training a group from another department, volunteering to speak at team meetings).

Practice it plenty of times alone, using the resources you will rely on at the event.

Engage with your Audience

Try to engage your audience. This makes you feel less isolated as a speaker and keeps everyone involved with your message. Ask leading questions aimed at individuals or groups, and encourage people to participate and ask questions, but only at the end. Avoid the words "just," "I think," "actually," Instead, say what things are, be clear and direct.

Pay attention to the way you are speaking: slow down by breathing deeply. Don't be afraid to gather your thoughts; pauses are an important part of the conversation, and they make you sound confident, natural, and authentic.

Never read word-for-word from your notes. Instead try to memorize what you are going to say, or use your cue cards when you need them.

Key Points

In order to become a better speaker:

- Plan appropriately
- Practice
- Engage with your audience
- Pay attention to body language
- Think positively
- Cope with your nerves
- Watch recordings of your speeches

If you become a good public speaker, it can help you get a job or promotion, raise awareness for your team or organization, and educate others. The more you force yourself to speak in front of others, the better you become.

Remember the proverb: "Rome was not built in one day."

Section A9: Competitions - Events and MATHeatre

Mathematics and competitions can be combined in many ways; the MATHeatre competition is one of them. In this chapter, we are going to lay down the guidelines on how to organize a competition or event.

I. Planning and Administration

A well planned event will save you time, resources and money. You should be able to break down the key roles and tasks of each of your team's members so that you can proceed effectively. Determine your target audience and if the competition/event is local, national or international. After identifying your audience, try to collect contact details (emails, addresses, etc.) in order to create a database that will help you to send invitations, information, promotions, etc. It should be noted that Decision Makers (ministers of education, school directors, national agencies, etc.) can play a very important role in the dissemination of your completion/event. If the number of the participants is great (more than 200 students), it is better to separate the completion/event in more than one phases.

II. Venue and date

Finding a venue and setting a date are probably the first major difficulties encountered when organizing a successful event. It is hard to continue with any other aspect of the overall planning until you overcome these two key hurdles. It is recommended that you examine these two questions at the same time: select an ideal set of dates and search potential venues in order to find the best possible fit.

It is crucial to pick a good date for your competition/event to avoid competing with other events in your area that will attract the same audience. For achieving the best possible scheduling, you should check that your event/competition date does not conflict with any other popular events. Additionally, you should take into consideration holidays and university and school calendars to avoid scheduling the completion/event during exams.

Choosing a venue is one of the most important steps in organizing a completion/event. A poor choice can undermine even the most well-planned events, while a good one can make a good event even better. When looking at a possible venue, you should take into consideration the potential cost. Be sure to check all venue costs (venue, security, catering, etc.) to verify that it will fit your budget. Additionally, make sure that it meets all your needs. For example, you would probably need a venue that has enough parking lots, a presentation room with projector and having a suitable size for your event. You should also take into account that if your event lasts more than one day attendance may vary, especially in week-ends, so you will have to manage your space accordingly.

III. Budget

It is the organizing team's responsibility to keep track of all event expenses. To begin planning your budget, firstly you should consider how many attendees you are expecting as this will have a direct impact on your choice of venue, supplies, food and equipment. As soon as you have a clear idea of the scale of your competition/event you are set to move forward. Even though each event might be different, you should be able to identify and break down your main costs. Indicatively, you should be able to estimate your expenses by taking into account the cost of the:

- Venue
- Food and Beverages
- Supplies and Equipment
- Marketing/Promotion
- Travel and Accommodation
- Gifts and Memorabilia

Additionally, where applicable you should aim to use volunteers in order to avoid hiring professionals for the tasks that do not require a high level of expertise. Also, a good way to tackle some of the costs is to find sponsors that will be willing to share some of the expenses.

IV. Promotion

Promotion is arguably the most difficult and time-consuming aspect of organizing an event. It is also extremely important, as it is in your best interest to promote your event in order to maximize attendance. This can be done in many ways with varying costs. You will be forced to be proactive, outgoing, and you should be ready to make some new connections. When promoting your event you should have a clear idea of your target demographic and try to focus your efforts via channels that are more accessible to them. The more variety and imagination you will use in your dissemination efforts the more rewarding will be the results. The use of social media is highly recommended as it is free, and it allows reaching an audience, which it might be otherwise inaccessible. Additionally, depending on the budget you should consider promotion through radio and television. You should also print posters and leaflets and distribute them in schools, universities, NGO's, etc. that might be interested in your event. In many cases, it is highly recommended to create a webpage or advertise through your organization's webpage, by providing specific information for the event (maps, cost of attendance, FAQ, etc.). You should remember that having a well-organized and fun website with rich content is the easiest way to convert traffic into registrations.

V. Speaker and Judges

A high-profile speaker is always a very good way to create buzz about your event. In some cases, it can also help you promote your event/competition and even sell tickets. Depending on your event you should cap the total allotted speaking time appropriately and manage your time effectively.

Having a panel of Judges to evaluate the final presentations is a great way to provide added value to your event. Similarly with key speakers, you should aim to have at least one or two high-profile judges, in order to add to the credibility of your completion/event and to serve as an extra mean of promotion.

PART B: MATHeatre AND MATHEMATICAL COMPETENCES

Mathematical Content and Examples

Integration of MATHeatre in the learning process

In the GENERAL REMARKS and PART A of these Guidelines, we presented thoroughly the advantages of using the theatrical approach in mathematics learning. We argued that MATHeatre is a motivation tool that promotes communication skills and improves mathematical learning. Additionally, we explained the various types of activities and the approaches for exploiting and linking MATHeatre in the curriculum. Finally, we analyzed the role of the teacher and the student as a director and the importance of the theoretical background in the proposed activities. However, it is quite clear that some examples will further support these ideas. For this reason, a number of supporting tools have been produced under the title ACCOMPANYING TOOLS/MATERIAL that are outcomes of this project and accompany the present manual. Subsequently, we have to ensure that these elements are in line with the responsibilities of a teacher.

These supporting tools provide numerous examples of practices in this area. Additionally, there are analyses and comments on many of these scripts or stories associating them to the areas of mathematics they are referring to, such us the age group of the pupils that are appropriate for and the pedagogical outcomes/ goals that can be achieved through them.

From the presentations in PART A it becomes obvious that the MATHeatre approach can be implemented as following:

(a) In theatrical performances that are implicitly supporting the mathematical curriculum

Such activities are formally prepared and usually are taking place:

- Through theatrical plays that are to be performed in an event in a school
- Through participation in a competition
- Through a specially designed performance in a class

(b) In performances that are explicitly and immediately supporting the mathematical curriculum

Such activities could be part of the everyday program in classroom and can be organized in simple terms with restricted use of demanding theatrical outfits, tools or materials. They can be prepared and presented:

- Through the adaptation or the preparation of a specially designed script by the teacher in order to enhance the learning of a concept, a process or other mathematical activity that is part of the syllabus, by taking into consideration the background of the pupils and the associated mathematical objectives.

- Through the adaptation or the preparation of a specially designed script by the pupils in order to enhance the learning of a concept, a process or other mathematical activity that is part of the syllabus, by taking into consideration the background of the pupils and the associated mathematical objectives. Obviously, this preparation should be under the supervision of the teacher (perhaps as part of the project).

Section B1: Examples/Illustrations of Using MATHeatre outside a Regular Mathematics Class

Example 1

Ask the participants to watch a video from the database of the Le-math project concerning competitions.

(a) Please analyse it according to the standards of the Analysis Book

(b) Please assess it according to the criteria for assessing the MATHeatre competition.

Example 2

Activity: Take part in a local MATHeatre competition, using an existing script.

- What are the preparatory steps for this participation?
- What are the expectations from such participation?
- How are you going to exploit this?

Example 3

Activity: ake part in a local MATHeatre competition, with a script you wrote, by using *Mathematical Stories* or by adapting an existing play.

- What are the preparatory steps for your participation?
- What are the expectations from such participation?
- How it could be exploited?

Example 4

Activity: Conduct research in order to identify a story related to the values of Mathematics. Based on this, develop a script for a theatrical play in the spirit of the examples used in the Manual of Good Practices.

Ask students to prepare a performance based on this script and present it to pupils of the appropriate age group in an afternoon set for extracurricular activities in the school.

After the performance, direct a discussion in order to give the opportunity to students to reflect on the moral and the aesthetics of the play, as well as other values that are presented and are relevant to mathematics.

Example 5

As a teacher, you want to help girls to get rid of their fear and repulsion of mathematics. In this context, presenting a theatrical play about Hypatia could be a good opportunity to help them address their concerns. In this context, you can develop a performance as part of the event for the Woman's Day. You can base this on the story of Hypatia and on the following photo which is part of the Raphael's painting "The School of Athens", depicting Hypatia among scholars of the ancient world.

Ask the pupils to prepare a script, as part of a project, and proceed to its presentation in a performance at the school. In this effort they can get information from many resources such as:

1. Eves, H. W. (1964). *"An introduction to the history of mathematics"* (5th ed.). New York, NY: The Saunders Series.
2. Grinstein, L. S. and Campbell, P. J., ed. *"Women of mathematics."* New York, NY: Greenwood Press.
3. McLeish, J. (1991). *"The story of numbers."* New York, NY: Fawcett Columbine.
4. Osen, L. M. (1992). *"Women in mathematics."* Cambridge, MA: The Massachusetts Institute of Technology.

"The School of Athens", by Raphael

Section B2: Examples/Illustrations of using MATHeatre in the Context of a Regular Mathematics Class

As already mentioned this approach can provide added value to the learning of mathematics. In view of this, the teacher has to do preparatory work in order to link MATHeatre activities to the topic he has to teach. For this, the following ideas can help:

Samples of action plans (level, number of participants, subject, time, preparation, process):

Sheet N°: ..... **Title**: *Solving Linear Equations*

Level: 5th / 12-13 years old.

Goals: Mathematical / pedagogical content: Understanding the technique of solving equations. Make the students feel in their movements the mathematical technique of the resolution of linear equations.

Length: 15 min / 1h

Participation: The whole class. The teacher chooses the actors. The other pupils are the audience. The actors can move alone or the audience can tell them what to do.

Where? In the classroom.

Material needed: the board, one chair (« = »), two colours of T-shirts (or dark/white clothes) or masks.

Pedagogical support: nothing or the video explaining the rules:
http://www.dailymotion.com/video/x6p7h8_mathematique_creation#UcFkydgric

What to do before? Explaining the rules of the game.

Procedure: The teacher writes an equation on the board and ask the pupils to be volunteers to play an "x" or a number. The pupils organize themselves to set in the equation and then solve it while moving.

What to do after? Do it again, increase the difficulty, let the students create their own equations to solve, find problems and how to solve them. After the game, the teacher makes the connection with the usual method of solving an equation.

Remarks: It is interesting for the pupils to participate as actors in this activity, because they can easily learn the method of solving equations with the movement of their body. The audience also benefits from the activity, as observing from distance helps them to visualize the mathematical technique. The teacher should be careful to involve as many students as possible. The members of the audience must switch with the actors and vice versa.

Variants: You may repeat the exercise after you increase the difficulty. Use different equations.

Blank sheets to be filled by teachers from their own experience. A file of these different experiences can be then easily created:

Sheet N°: .....

Level: .....

Goals: .....

Length: .....

Participation: .....

Where? .....

Material needed: .....

Pedagogical support: .....

What to do before? .....

Procedure: .....

What to do after? .....

Remarks: .....

Variants: .....

The following are examples of approaches used by participants for the MATHeatre competition preparation.

Example 1

The "success story" of the 3rd place in the MATHeatre Competition 2014:

The Prime Kingdom

3rd price in MATHeatre competition 2014, category 9-13,
ZS Fr. Plaminkove School, Czech Republic

Preparation

Teacher trainees are introduced to the concept of the competition MATHeatre Teaching and Learning Mathematics through Mathematics Theatre. They discuss the ways in which mathematics can be made more interesting and entertaining to pupils and discuss the idea of the proposed methodology.

Realization

Teacher trainees are introduced to the teaching unit The Prime Kingdom. It was conducted in the following stages.

Stage 1: Two sessions of mathematics in English (CLIL) in which pupils were introduced to the concept of prime numbers, Eratosthenes' sieve, prime twins and emirps. In these two lessons the pupils learned the essential vocabulary and mathematical concepts.

Stage 2: One session: The teacher introduces the concept of theatre play about prime numbers and introduces the basic plot (a prince must choose a princess, the princesses try to solve tasks about prime numbers, the prince will marry the princess who will solve correctly the most tasks). The students are invited to come

with possible characters and their role in the story. The goal is to make every pupil involved and to let them develop the basic plot. (Pupils – with teacher's help – come with characters such as counselors, maids, queen, narrator etc.).

Stage 3: Five sessions: Pupils develop and rehearse the play.

Teacher trainees are shown video recording with *The Prime Kingdom*.

The Scenario

King Prime the Second decides it is time for his son Prime the Third to get married. He invites two princesses (Factoria and Compositia) and their maids to the castle and gradually gives them three tasks about prime numbers. There are two counselors who do not want the prince to get married as they want the throne for themselves. Therefore they try to tell the princesses wrong answers and prevent their victory. The tasks are:

1. *How many prime numbers are there between 1 and 50?* (The counselor suggests a wrong answer to one of the princesses. The right answer is provided by Compositia's maid, who uses Eratosthenes sieve to solve it).
2. *How many prime twins are there between 1 and 50?* (The other counselor suggests a wrong answer to Compositia. The right answer is provided by Factoria's maid. Again she uses Eratosthenes' sieve to solve it plus the additional knowledge of the difference of value 2).
3. *How many emirps are there between 1 and 50?* (The right answer is provided by Compositia's maid. Compositia answers and appears wiser. The prince sees how smart she is and he falls in love with her).

The play finishes by Compositia's victory and prince's proposal to marry her.

Post-task

Teacher trainees discuss the video with respect to

- Mathematical content
- Presentation
- Language

They work in pairs to propose possible improvements to the play.

They develop a lesson plan in which they will make use of the fairy tale *Prime Kingdom*. What would precede, what would follow?

Follow-up

Task for teacher trainees – The setting of a fairy tale is also suitable for the introduction and practice of many other mathematics concepts. Use the same scenario but propose other possible mathematics content.

Example 2

Second Prize of the Le-Math script competition:

Geoland

Written by Marilena Vilciu and Theodor Draghici from Romania

Refer to the Analysis of play Geoland, page 10 of the manual of scripts for MATHeatre, to identify whether it is of any use in your lessons.

Comment on the usefulness of this analysis to your teaching by identifying, in particular:

(a) What do you consider helpful and why?

(b) What do you consider not helpful and why?

Then go to the actual script in the Manual and study it. To what extend does this reflect the analysis you read as well as the comments in (a) and (b)?

What arrangements are you going to make in order to use this in your classes (during a lesson)?

Example 3

The Magician

A theatrical play for demonstrating the "magical powers" covered behind mathematical processes and concepts.

This play has been designed so that:

(a) It can provide motivation for the study of mathematics

(b) It can provide the background for comprehension and realization of the need for factorization into primes in the set of Integers

(c) It can provide the forum for discussion of remarkable properties of numbers and opportunity to refer to their role in the history of the civilization

(d) It can demonstrate the process of problem solving and provide some aspects leading to it

(e) It can provide opportunities for reflection on the values of mathematics

(f) It can demonstrate the value of reflection and reasoning through mathematical activities.

Characters:

The Magician: A person wearing a tall hat

Andrew: A pupil, 12 years old

Mary: A pupil, 12 years old

The Teacher: A lady dressed formally for teaching, around 35 years old

ACT I

SCENE 1

In Andrew's bedroom, in the afternoon, he is sitting in front of his desk looking into an open book. Mary is also there sitting on a chair opposite to him.

Andrew: What on earth does it mean of a prime number? And why do we have to find out its meaning? Do you think that it is of any use to know about it? Ok, I understand that it is useful to have a knowledge of division as we use it in order to divide, say 12 candies to 3 persons. But what is the use of a prime?

Mary: You are right. This is another idea of mathematicians to torture us.

Suddenly a person, The Magician, enters into the room with a triumphal style.

The Magician: I am a magician, I can demonstrate to you that I can read your thoughts without you revealing anything.

Andrew and Mary: You are kidding! This is impossible! You are out of your mind to say such a thing. There are no such creatures in real life, but only in fables.

The Magician: Wait a minute and I can prove it to you.

Andrew and Mary: How?

The Magician: Think of a 3-digit integer and repeat of what you thought again by writing it next to the original number, so that you form a 6 digit number. For example if you have thought 352 then the 6-digit number is 352352.

Andrew and Mary: Ok we are done!

The Magician: Now divide this 6-digit number by 7. You can use your calculator if you like to accelerate the operations.
(a small pause to give them the time to do the calculation).
I claim that the quotient you found from this division is an integer. Am I right?

Andrew and Mary: *(uneasily and being a little embarrassed)*
You are right.

The Magician: Now divide this quotient that you have just found by 11. *(a small pause to give them the time to do the calculation).*

I claim that the quotient you found from this division is an integer. Am I right?

Andrew and Mary: *(uneasily and being more embarrassed)* You are right.

The Magician: Now divide this quotient that you have just found by 13.

(a small pause to give them the time to do the calculation).

I claim that the quotient you found from this division is an integer. Am I right?

Andrew and Mary: *(uneasily and being a little embarrassed)* You are right.

The Magician: Furthermore I claim that this last quotient you found is the 3 digit number you thought initially. Am I right?

Andrew and Mary: *(uneasily and being amazed)* You are right. But how can you guess?

The Magician: I told you, I am a magician and I can read your thought.

ACT II

SCENE 1

The next day in the classroom the two pupils are sitting amazed and discussing the experience they had during the previous day with The Magician.

Andrew: Mary, I cannot understand how that man yesterday could guess all these details without any of us revealing anything to him. Do you think that he is really a magician?

Mary: I cannot either. Possibly some people may have this charisma.

The Teacher: Andrew and Mary what are you talking about?

Andrew and Mary: Oh, madam ... Yesterday when we were studying all of a sudden a magician enter the bedroom and we had the following experience.

SCENE 2

The magician enters into the room in a sudden way. The two pupils repeat the dialogue they had yesterday with the magician.

The Magician: I am a magician, I can demonstrate to you that I can read your thoughts without you revealing anything.

Andrew and Mary: You are kidding! This is impossible! You are out of your mind to say such a thing. There are no such creatures in real life, but only in fables.

The Magician: Wait a minute and I can prove it to you.

Andrew and Mary: How?

The Magician: Think of a 3-digit integer and repeat of what you thought again by writing it next to the original number, so that you form a 6 digit number. For example if you have thought 352 then the 6-digit number is 352352.

Andrew and Mary: Ok we are done !

The Magician: Now divide this 6-digit number by 7. You can use your calculator if you like to accelerate the operations.
(a small pause to give them the time to do the calculation)
I claim that the quotient you found from this division is an integer. Am I right?

Andrew and Mary: *(uneasily and being a little embarrassed)*
You are right.

The Magician: Now divide this quotient that you have just found by 11. *(a small pause to give them the time to do the calculation)* I claim that the quotient you found from this division is an integer. Am I right?

Andrew and Mary: *(uneasily and being more embarrassed)*
You are right.

The Magician: Now divide this quotient that you have just found by 13.
(a small pause to give them the time to do the calculation)
I claim that the quotient you found from this division is an integer. Am I right?

Andrew and Mary: *(uneasily and being a little embarrassed)*
You are right.

The Magician: Furthermore I claim that this last quotient you found is the 3 digit number you thought initially. Am I right?

Andrew and Mary: *(uneasily and being amazed)*
You are right. But how can you guess?

The Magician: I told you, I am a magician and I can read your thought.

SCENE 3

The Magician leaves the classroom. The teacher smiles and then starts questioning.

The Teacher: Andrew can you tell me what is the issue, that is what is the problem that we face at this moment?

Andrew: Madam, do you mean that we face a mathematical problem? I do not understand that this is the case.

The Teacher: Yes, indeed. What is actually the first step in the process of problem solving?

Mary: Understanding the problem. But where is such an issue. We do not have data and we do not have results that we are looking for.

The Teacher: Andrew, do you agree that we do not have data?

Andrew: I think we have some information but I cannot see how to proceed.

Mary: Oh madam, We have as data the three numbers that we use for divisions, that is 7,11 and 13.

The Teacher: Is that all the information that you have? How did the magician start his demonstration?

Andrew: I see. He considered a 3-digit number.

Mary: And then he asked us to repeat this number, thus forming a 6-digit number.

Andrew: And then we started dividing this 6-digit number consecutively by 7, 11 and 13.

Mary: And we observed that at each step we got a quotient that was an integer and finally we reached the initial 3-digit number.

The Teacher: So what is the problem here?

Andrew: The Question is: Why, by taking a 3-digit number repeating it again to form a 6-digit number and then dividing consecutively by 7,11 and 13, we always have perfect divisions and we finally reach the initial number we thought?

The Teacher: That's perfect. Now what are the important aspects of our information?

Mary: The facts that:

(1) we repeated the 3-digit number to form a 6-digit one.

(2) We divided this consecutively by 7, 11 and 13.

(3) We reach the point we started from.

The Teacher: Fine! I hope that everybody is aware with the various concepts involved and that you have understood the problem. Now what is the next step in approaching the problem.

Andrew: Devising a plan, but I cannot see anything that could help me.

The Teacher: Let me give you a hint. If you have the number 24 and you divide it by 2 and then by 3. How could you get the same result with only one division? And what is the relation of the original number with the outcome and the divisors?

Mary: Obviously by dividing by 2 times 3 that is by 6. Oh I see the plan is to consider the product of the numbers 7, 11 and 13.

Andrew: Which is 1001 and then the product of 1001 times the original 3-digit should be the 6-digit number.

Mary: It is obvious which our plan is. Let's move to the next stage of Implementing the plan.

Andrew: Eureka! Eureka! If you multiply a 3-digit number by 1001 you get a 6-digit number which is the one that we can form by repeating in a line the given 3-digit number.

The Teacher: Can you see now the solution to the problem?

Mary: Yes, the magician was just using this last property mentioned by ANDREW, and then he was doing the inverse operation of multiplication, that is division, and then instead of doing the division by 1001, he was repeating the process by consecutive division by 7,11 and 13.

The Teacher: Now let's move to the next step of problem solving, that is to review and investigate what we have found. Does the process work for every case and why?

The play can continue in this manner by adding dialogues for other scenes depending on the objectives of the syllabus. For example:

(i) Elaboration of the divine properties of numbers

(ii) Elaboration on the prime factorization and its properties etc.

Example 4

The Pythagorean Theorem

A very important topic that is included in every mathematics curriculum is the Pythagoras Theorem. This topic is of great interest, as it has a very broad range of applications. It serves as a link between various areas of mathematics (geometry, number theory, algebra, trigonometry) and it is a very important part of the history and culture of the human civilization and the history of mathematics. Consequently, presenting it through a theatrical play offers many advantages in the process of learning mathematics. The example that follows presents a script that can be used in any mathematics class. It should be noted that you might find many other theatrical plays with this topic that can be used in the classroom.

Characters:

Mr Nikos (Math teacher)

Vasily (Foreman)

Kostas: (Coffeehouse keeper)

Builder aides A and B

Students A, B, C

Men at coffeehouse (silent roles)

Students (in class, extras)

SCENE I

Mr Nikos, Kostas, Vasily, customers at coffeehouse.

(At a local coffeehouse. A few customers chatting, others playing backgammon. Mr Nikos, the high school teacher, steps inside and sits at a table.)

Mr Nikos: (*to the coffeehouse keeper)* Mr Kostas, can I have a coffee please? (Unfolds the newspaper on the table and reads. Shortly after, Mr Kostas brings the coffee) Mr Kostas, do tell me. Does Master-Vasily, the foreman, come to the coffeehouse every day?

Kostas: Indeed he does, Mr Nikos. He'll be here any minute now. You're just in time to see him.

Vasily: *(steps inside and greets everyone)* Good evening folks!

Mr Nikos: Master-Vasily, welcome! Won't you come sit with me? There's something I'd like to talk to you about. I'll buy you coffee.

Vasily: I'd be glad to, Teacher! What brings you to our neighborhood?

Mr Nikos: Master-Vasily, I noticed today that you brought some tools to the schoolyard and that you also put up a fence in a far corner of the yard.

Vasily: That's right! Did you notice already?

Mr Nikos: Of course I did. So, I wanted to ask, what are you going to build?

Vasily: How did you know we were building something?

Mr Nikos: I sort of heard it through the grapevine and if it is so, I want you to help me with my next class.

Vasily: Anything, Mr Nikos! Always at your service. Well, we've been hired to put up a shed.

Mr Nikos: Great! Let me ask you something. How are you going to carve the shape of the shed on the dirt floor? Are you using any instruments?

Vasily: No, Mr Nikos. It's a simple task. We'll carve it out the old way.

Mr Nikos: Very well. That's exactly what I was hoping for. But tell me, do your aides know how to do it?

Vasily: Bah, I don't think so; they're too young to know.

Mr Nikos: Here's what we're going to do. You tell them to start carving the shape on the dirt floor and in the meantime I'll get there with my students for class. What would be a good time, do you think?

Vasily: Eight o' clock tomorrow morning.

Mr Nikos: Alright, then. We'll be there around 8:15. That will give me enough time to prepare them. Agreed?

Vasily: I'll be waiting for you.

End of Scene I.

SCENE II

Vasily, Aides A and B, Mr Nikos (high school teacher), Students A, B,C and other students (extras).

(At the schoolyard where the shed will be put up. The two aides gather their tools and put their gear in place. Planks, a few iron rods, ropes, a measure, nails etc. Master-Vasily enters).

Vasily: *(to his aides)* Hey guys, are we set?

Aide A: Yes, master-Vasily, we're set.

Aide B: Good to go! Just tell us what to do.

Vasily: Alright, listen up. I want you to try and carve the shape of the shed on the dirt floor. We'll build it there, in the corner. Just remember, it's got to be at a three-meter distance from the boundaries.

Aide A: Will do, Master-Vasily. *(Vasily exits for a while).*

Aide B: *(to the other aide)* Hey, George. Do we know how to carve out a right angle?

Aide A: We could, I guess, if we had a right angle – even a small one!

Aide B: But still, how could we have pulled off a right angle using a tiny instrument?

Aide A: So what are we going to do?

Aide B: We'll wait for Master-Vasily to come and we'll ask him. It's not a shame to admit we don't know how it's done.

Aide A: True. After all, until now either the surveyor or the engineer would place the marks using a measuring instrument.

Aide B: Let's just wait for the master.

(Master-Vasily enters with Mr Nikos and his students)

Vasily: How does it go, guys? Are you making any progress?

Aide A: Master-Vasily, we didn't do anything; we didn't know how.

Aide B: Yes, till now, the contours were done by either the surveyor or the engineer.

Vasily: Do you mean to say you've never heard of the three-four-five method?

Aide A: No.

Vasily: Alright, listen up. You'll take a piece of long thin rope and use your measure to tie four successive knots into it. One at the beginning, one at three meters, another one at four meters and the last at five. At the corner which lies at three meters from the boundaries you will place a large nail or a peg onto the second knot of your rope and hammer it into the ground.

Aide A: And then?

Vasily: Then you will spread the knotted rope along the two sides of the boundaries at three and four meters, place pegs on the knots and connect the two pegs with the five-meter rope. *(The aides do what the foreman tells them and realize that they have a perfect right angle)*.

Aide B: Master-Vasily, we did it!

Aide A: Unbelievable!

Mr Nikos: Children, did you see what just happened?

All: Yes, sir.

Student A: How is this possible?

Mr Nikos: Oh, it's possible alright!

Student B: And it works only with three, four, five?

Mr Nikos: No. It works with all multiples of three, four, five.

Student C: And why is that, sir?

Mr Nikos: Well, it is a mathematical theorem. But we'd better discuss this in class. Come on! *(They exit the stage).*

End of Scene II.

SCENE III

Mr Nikos (high school teacher), Students A, B, C and extras (students).

(A school class. The children step in with their teacher; they take their seats.).

Mr Nikos: So what do you think, guys? Did you like the demonstration of Master-Vasily and his aides?

All: Yes, very much!

Student A: But sir, not all of us had a clear view outside; can we repeat it here so that we'll know for sure how it's done?

Mr Nikos: Of course we can. This is exactly what I had in mind, that's why I've brought everything we're going to need. *(He walks behind his desk and picks up a 60X60 cm piece of plywood, one meter of string, a hammer and nails)* Alright now, let's repeat the measuring process.

Students A and B: *(Approaching the teacher's desk)* What are we going to do now, sir?

Mr Nikos: First, you will tie a loop into one end of the string and a second loop at precisely 40 cm from the end. Then, you will pass a nail through each loop.

Student B: *(The kids measure and place the nails)* Done, sir.

Mr Nikos: Hammer the two nails into the wood whilst keeping the string taut more or less in parallel to one side.

Student A: Done!

Mr Nikos: Now, along the same piece of string, at precisely 30 cm, tie a loop then pass a nail through it. Create another loop at 50 cm.

Student B: All set.

Mr Nikos: Pass the end loop through the first nail then pull the other until the string becomes very taut.

Student A: There it is.

Mr Nikos: Now hammer the nail keeping the string well-stretched.

Student B: This is fantastic! It looks like a perfect right triangle!

Mr Nikos: It not only looks like one, it is a perfect right triangle! Lift the plywood for everyone to see.

All: Yes, it's incredible!

Mr Nikos: Does anyone know who Pythagoras was? *(students raise their hands)* Go on, Yiannis.

Student A: Yes sir, he was an ancient philosopher.

Mr Nikos: Anyone cares to add something? *(again, students raise their hands)* Yes, Marios.

Student B: Sir, he was also a mathematician.

Mr Nikos: Anything else?

Student C: Yes, sir! He was a musician too!

Mr Nikos: Very good. Does anyone know where Pythagoras hailed from?

Student C: Yes, sir. He hailed from Samos.

Mr Nikos: Indeed. That's why he is known as "Pythagoras of Samos" – for some, one of the Seven Sages of Ancient Greece.

Student C: And where does Pythagoras fit in with this story, sir?

Mr Nikos: You see, when he was young, Pythagoras traveled to Egypt, where at the time a great civilization had developed. So, among the many things he saw there, was the Egyptian rope, the harpedone.

Student A: What's that, sir?

Mr Nikos: It was a measuring tool, namely a rope with twelve equally spaced sections marked by tied knots and nails. So using this rope, the harpedone, the ancient Egyptians could carve out a right triangle, just as we did today. In other words, the same method was used by the Egyptians since 3000 BC, who claimed, 2500 years before Pythagoras, that the angle formed by the 3 and 4 meter sides was in fact right.

Student B: It had a weird name, this Egyptian rope.

Mr Nikos: Harpedone is the name of this simple tool; and the harpedonaptae were those using it to carve right angles on the dirt floor. It is said that this very method was applied for the construction of the Pyramids. The Indians and the Chinese went on to emulate their example.

Student C: How come this very old story relates to Pythagoras?

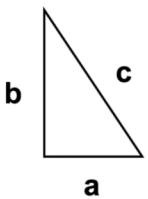
Mr Nikos: Because in the 6th century BC, Pythagoras (569-500 BC) and his students provided a proof of the claim, in other words, that the angle where the three-and-four meter sides meet is a right angle. Therefore, the equation is known in the history of mathematics as the Pythagorean Theorem.

All: Unbelievable stuff!

Mr Nikos: So, have you ever heard of the Pythagorean Theorem?

Student B: Yes, sir, I think we have.

Mr Nikos: And what does it say, the Pythagorean Theorem? That "in a right-angled triangle, the sum of the squares of the two vertical sides is equal to the square of the hypotenuse". *(On the blackboard he draws a right triangle with a, b, and c sides)*.



Therefore, if a=3, b=4 and c=5 we shall see that:
$3^2=9$, $4^2=16$ and $5^2=25$, and obviously 9+16=25

Student A: Does this work only with 3, 4, 5?

Mr Nikos: Of course not. The same applies if we double these three numbers into 6, 8 and 10. We can see that their squares are 36, 64 and 100, and that 36+64=100. In fact, it works with any multiples of these numbers because of the validity of this equation: $a^2+b^2=c^2$.

Student B: And how can we actually demonstrate a proof of the equation?

Mr Nikos: Today, proof of the Pythagorean Theorem can be provided in many ways, depending on the students' ages and math knowledge. As for us, we shall demonstrate a rather simple proof.

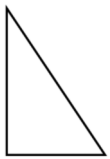
Student C: Sir, can I come up to the blackboard?

Mr Nikos: Yes, why not. Come on up, Constantinos.

Student C: *(Stands in front of the blackboard and picks up a piece of chalk)* All ready, sir.

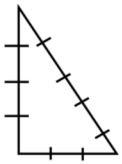
Mr Nikos: Now draw a right angle and try to give it sides that equal 3, 4 and 5 units.

Student C: *(He draws the triangle)* Ready, sir.



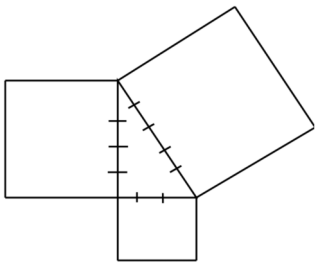
Mr Nikos: Now divide each side into 3, 4, or 5 parts depending on their length.

Student C: *(Divides the sides accordingly)* Now?

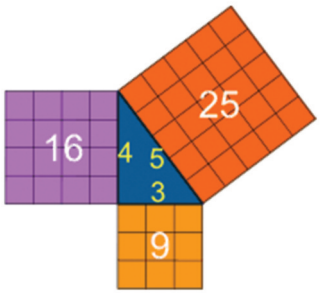


Mr Nikos: Now you will draw a square on each side.

Student C: *(He draws the squares)* Alright.



Mr Nikos: Now draw parallel lines from the points where you had divided the sides. Do the same along the vertical side of each square.



Student C: *(He draws the lines)* We have created several small squares.

Mr Nikos: Now count those "small squares" as you call them.

Student C: They are 25 on the hypotenuse side and 16 and 9 respectively on the two vertical sides.

Mr Nikos: And what do we see?

All together: *(In unison)* That the 25 small squares of the hypotenuse equal the sum of 16 and 9 squares of the other sides.

Student A: As simple as that?

Mr Nikos: Exactly, as simple as that! Of course, there are several proofs available, depending on the students' age or math knowledge. Do you realize how useful this theorem is, how practical it used to be? And how it is to this day applied in the construction field?

Student B: Yes, sir.

Student C: We should have more lessons like this one!

Student A: Now there's no way we'll ever forget the Pythagorean Theorem!

Mr Nikos: *(In the meantime, the bell rings)* Thank you, children. God bless. You may go now.

The end.

EXAMPLE 5

Methodology of Problem Solving

How to solve a Problem exploiting math-theatre

The systematic approach we follow for solving a mathematical problem is similar to the one that we use for any problem. On this matter, the Hungarian mathematician, George Pólya suggested a four-step method that facilitates problem solving and can be used in the classroom to enhance the students' problem solving abilities. This approach can be explained by making reference to a situation or a problem that the students' can identify with or are familiar with and then gradually apply it to a mathematical problem. For this we can ask a team of students (or all the students depending on their skills and levels of abilities) to write a script that will have three stages:

Stage1: Write a scene based on the following story. The chief of staff of a certain country has been given instructions to organize a military operation in order to eliminate some installations that endanger the well-being of the people of his country. The following pictures could direct you towards the actions and steps that he is planning to take in order to address the problem:

What do these activities remind you of?	
 	Collecting Information
 	Developing an Action Plan

 	Materialising the Action Plan
	Assessing the Outcome of the Campaign

The script is expected to contain dialogues that will illustrate how each of these activities could be carried out. The students are requested to devise questions and present ideas that will help them in reaching the goals of each step.

Stage 2: Write a scene that is based on a mathematical problem set by the teacher. The approach for its solution could have analogies of the activities proposed in Stage 1. The emphasis for each step is to present questions, discussions, dialogues or assertions in a similar manner with Stage 1.

Stage 3: Write a theatrical scene involving a team of students discussing the similarities of the two approaches in the two previous scenes.

Finally, the teacher will proceed in asking a group of students to present a play based on the scenario they just developed.

After the performance, the teacher can open a discussion about what the students learned out of the whole activity and stress the important steps in approaching a mathematical problem.

The following problem can be suggested as the basis for the preparation of a script at Stage 2 (This problem is suggested, because it can be used in the learning process of students by a wide range of ages. It can be used at the primary level for learning

the basic arithmetic operations, but also at the upper classes in high school for learning basic concepts in the Theory of Numbers).

A group of fanatical followers of a religious sect, using the information of their written texts and the calculating power of computers concluded that doomsday will come during the year when the first day of one of the following centuries, falls on a Sunday. With this in mind, what is going to be the year at which the world will come to an end?

Hints that can be included in the development of the script for Stage 2	
STEP 1 **Understanding the problem**	What is required by this problem? Do we understand all the phrases/concepts that we meet in the problem? What are the data and what are the expected results? Do you know how we determine the beginning of a century? For the present problem we adopt as the beginning of a century the year where the last two digits are 00. Do you know how we determine a leap year according to the Gregorian Calendar? Do you know that the 1st January 2000 was a Saturday?
STEP 2 **Developing a plan**	An important element to consider is what years are leap and what are not. Taking this into consideration can we find what day is the 1st January of the year which is the starting point of a century? How useful is going to be in this process the name of the day of the 1st January 2000?

STEP 3 **Accomplishing the plan**	Proceed in finding the possible names of the days of 1st January of the years at the beginning of a century, that is 2000, 2100, 2200, and so on.
STEP 4 **Verify/check/ review/generalize**	Check the rationality of the outcomes. Can you think of different approaches for the solution?

Basic Tips that can help the students in development of the scenario, the preparation of the script and the acting of the play

1. Ask them to start with a character. It is useful to set the context by asking them to identify the characteristics of the main character of the play, his/hers personality and role in the play.
2. Ask them to define other characters (with secondary role) in the play and to identify their characteristics.
3. Ask them to set the scene and connect the whole act with mathematics
4. Develop the various activities, dialogues, discussions that constitute the content of the play.
5. Ask them to discuss and include in the play description of the scenery.

EXAMPLE 6

A Mathematical Investigator

Staging: One hat and one trench coat could be used for the pupil who will play the role of the inspector.

Pupils often feel lost under the enormous amounts of knowledge they have to absorb. All this knowledge is frequently mixed in their heads, and even if they succeed in memorizing all the definitions and properties, they have some difficulties to identify and apply them in the context of a mathematical demonstration or reasoning.

Several math problems can be explained, solved and written thanks to this theatrical activity, and it can surely help students to improve their logical reasoning and their ability to synthesize information.

A mathematical demonstration can be compared to a police investigation. The mathematician who tries to find a solution to a problem is like a detective. In this context, the teacher can introduce to his class a recurrent character: a mathematical investigator.

He has to make some observations

- He has to locate in the text what he has to prove, while reading carefully the data of the exercise. Sometimes he knows exactly what he has to prove (demonstrate that this quadrilateral is a parallelogram) and some other times he has to guess (what is the nature of this quadrilateral?).

- He also has to identify the useful information among others in the data of the exercise.

The investigator can be assisted by other characters who can be either “witnesses” or “experts”. These characters will try to help him during his investigation, by guiding him through the contents of the text and reminding him of the math content he is supposed to know.

A mathematician making a proof is like a police investigator: he has (given):

- the clues he can observe (the information given in the problem)
- his knowledge, what he learnt in class(definitions, properties, theorems)
- his experience (memories of resolution of ancient similar problems)
- his instinct (that can be helped by tools like schemas)

The questions usually are:

- What do I have to do? Is the question clear or do I have to guess?
- What do I have in my disposal?
- What do I know about it? What link can I make between this and my knowledge (key words to identify)?

Then, by making connections between observations, knowledge and the conclusion guessed, the student can make the proof, in an organized and logical way of thinking.

Writing the report

Pupils are often demotivated when they see their teacher's correction or answer written on the board. That is because there is no written trace of the thinking process that leads the educator to the solution of the problem. Students can only see the final written trace and many of them might think that the solution appears easily and immediately to the teacher. This approach might make them feel frustrated and that they are unable to do the same on their own.

They also do not understand why a comprehensive writing report is necessary: “I’ve found the answer to the question, why do I have to write all that?”

When the investigation activity is over, the police investigator has to “file” his report and “build his case”.

The mathematical investigator has to be clear and precise in his way of writing, like a police investigator, in order to be understood and accepted without any doubt.

If the investigator has to explain his proof alone, he may do it by writing a MATHFactor script. If he needs more characters, it can be categorized in a MATHeatre script.

Example:

PROBLEM: Points A and B are respectively the symmetrical points of C and D in the symmetry with centre O.

QUESTION: What is the nature of the quadrilateral ABCD?

Mathematical Investigator	**Police Investigator**
What do I have to find, what am I asked? By reading the text, I'll find the question in the problem: I have to find the nature of the quadrilateral ABCD. If I draw a diagram, I can guess that ABCD is a ... ? The answer is not in the question, I will have to guess it! For help I can underline the keywords: "symmetrical points" and "quadrilateral".	Who is the murderer?  Clues: Instinct of the investigator.
What do I know about these words? By reading the text again, I have to think of "central symmetry" and "quadrilaterals". I know that if A is the symmetrical point of C, that means that O is the midpoint of the segment [AC]. Therefore O is the midpoint of [AC] and with a similar argument it's also the midpoint of [BD]. Observation: [AC] and [BD] are the diagonals of the quadrilateral ABCD.	Witnesses said that ...  I know that ...

I have to find a link between my observations and my instinct and my knowledge/experience. I know that if the diagonals of a quadrilateral bissect each other, then the quadrilateral is a parallelogram. Do I have that? Yes! The diagonals [AC] and [BD]bissect each other at O, so ABCD is a parallelogram!! Observation: [AC] and [BD] are the diagonals of the quadrilateral ABCD. Problem solved!	Eurêka!  Problem solved!
File a comprehensive report. Given: A and B are respectively the symmetrical points of C and D in the symmetry with centre O, therefore O is the midpoint of [AC] and [BD]. Thus, we have: O is the midpoint of [AC] and [BD] which are the diagonals of the quadrilateral ABCD. But, as we know, if the diagonals of a quadrilateral bissect each other, then the quadrilateral is a parallelogram. Therefore ABCD is a parallelogram.	Police report 

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Figure 3: See Muniglia, M. (1994). *Le théâtre au service de l'algèbre au collège*. Repères N°16, Juillet 1994, Irem de Lorraine. Pupils from Collège Guy de Maupassant/Fleury/Andelle. Available from http://www.dailymotion.com/video/x6p7h8_mathematique_creation#.UcFkydgriZc. [Retrieved July 2, 2014.]

ACCOMPANYING TOOLS/MATERIAL

In the process of adopting the MATHeatre approach the user can find a broad range of examples that can be of great help either for approaching a particular area of the mathematical curriculum or for enriching his/her lesson or finding ideas for participation in competitions or preparing a theatrical performance for a particular occasion relating to mathematics. The present project has prepared some packages of such examples and are provided as part of its outcomes. The user can exploit these tools/material in order to enrich his/her store of resources. These tools/material are organized as following:

MT-Tool 1: Le-MATH Manual of Good Practices
(link to www.le-math.eu)

MT-Tool 2: Sample video of MATHeatre Plays
(DVD and link to www.le-math.eu)

MT-Tool 3: Manual of Scripts for MATHeatre
(publication and link to www.le-math.eu)

MT-Tool 4: Mathematical Stories for Theatre
(publication and link to www.le-math.eu)

ANNEXES

ANNEX 1 - Manual of Scripts for MATHeatre Analysis

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1. Fivepartacus

Manual of Scripts for MATHeatre: page 7

Math Topic: Roman numerals

Age Group: 9-13

Knowledge Background Required: Basic knowledge of arithmetic, knowledge of Roman numerals.

Knowledge Acquired: Consolidation of the notation of Roman numbers. Hints to remember the signs **V**, **M** and **Ṽ**. To learn that ¯ means multiply by 1.000.

Skills Acquired:

The preparation and presentation required for this MATHeatre play develops Numerical and Symbolic Comprehension for pupils: the understanding of the Roman numerals and the sign for multiplying by 1.000 is delivered in an amusing play enabling an easy understanding of the problem and helping on memorizing Roman numbers.

The students are informed about the Roman numbers one to five. The play leads students into a strange situation using perfect school slang and then the audience is brought back to the mathematical problem.

Numerical and Symbolic Computation is needed to understand the problem.

Visualization skills are developed as the Roman numerals are fixed onto the costumes of the actors.

Use and applicability: It can be seen that the understanding of this problem is easy using a script like this. Fun in mathematics combined with learning is the main task of this play. It is easy to use and can be rehearsed with each class, even in integration and special needs classes.

Preparing the problems, the presentation with the appropriate scenario, acting and the use of visual tools develop the Communication skills of the pupils.

2. Geoland

Manual of Scripts for MATHeatre: page 10

Math Topic: quadrilaterals, polygons

Age Group: 9-13

Knowledge Background Needed: quadrilaterals.

Knowledge Acquired: mathematical properties of particular quadrilaterals.

Skills Acquired:

Through a tale the students discover the properties of rectangle, trapezoid, rhombus. In this case, students can approach mathematics with a very attractive story like a princess - Square - makes the best choice of husband... the parallelogram.

Understand geometry through stories.

3. An outcast for a blueblood

Manual of Scripts for MATHeatre: page 14

Math Topic: Basic properties of rational and irrational numbers, philosophy of mathematics

Age Group: 14-18

Knowledge Background Needed: Description of basic theorems in elementary number theory, and Pythagora's theorem, the History of the calculations are needed.

Knowledge Acquired: Deepening of understanding the properties of irrational numbers.

Skills Acquired:

Comprehension: The realizations of the topics dealt with are; interdependent, mutual links of different domains like history of mathematics in different cultures, theoretical and practical computation aspects are developed.

Numerical and Symbolic Computation for calculations and properties of the natural, rational and irrational numbers.

Use and applicability: The story invented by the author leads to a deep mathematical understanding, and the presentation is suitable for increasing the real understanding of real mathematics

Communication (mathematics communication): Description of concepts and formulation of properties is developed in a very original way, by personalizing the numbers, and creating a real dramatic situation around the relation between the personages.

4. It is the story that matters, not just the ending

Manual of Scripts for MATHeatre: page 22

Math Topic: Reasoning about learning mathematics

Age Group: 9-13

Knowledge Background Needed: Ideas about learning mathematics, the reasoning in mathematics.

Knowledge Acquired: Deepening of understanding the reasoning, and logical arguing, deduction.

Skills Acquired:

Comprehension: Useful phrases and how to be convincing when you argue.

Numerical and Symbolic Computation in Logic are developed.

Use and application: To attract low-achievers.

Communication (mathematics communication): Description of everyday situations and finding the mathematics behind.

5. A Letter to Ms MacNamara

Manual of Scripts for MATHeatre: page 26

Math Topic: Complex numbers

Age Group: 14-18

Knowledge Background Needed: Square root, negative numbers.

Knowledge Acquired: Properties of imaginary unit.

Skills Acquired:

The preparation and presentation required for this MATHeatre play develops Numerical and Symbolic Comprehension for pupils: the understanding of power of imaginary units. And also develop Numerical and Symbolic Computation by expressing the result with the help of the residual classes of power.

They learn that Problem solving is an important part of Mathematics.

Use and applicability – scenario presents a new result, not typically use in the school's mathematics.

Preparing the problems, the presentation with the appropriate scenario and acting develops the Communication skills of the pupils.

6. A mysterious number

Manual of Scripts for MATHeatre: page 30.

Math Topic: Geometry

Age Group: 14-18

Knowledge Background Needed: Geometry, what constitutes proof vs conjecture.

Knowledge Acquired: steps followed to test a theory, properties of regular polygons.

Skills Acquired:

Analytical Thinking: proving theorems, conjectures.

Numerical and Symbolic Computation: generalization.

Problem solving: step by step solving, generalization.

Visualization: use of GeoGebra to show polygons and properties.

Communication (mathematics communication): mathematics in everyday life, real life scenario.

7. The logic of the stolen iPod

Manual of Scripts for MATHeatre: page 42

Math Topic: Mathematical Logics

Age Group: 14-18

Knowledge Background Needed: Work with sets, quantors, and basic rules of Logic algebra.

Knowledge Acquired: Work with simple and complex logic expressions, skills to apply quantors, main formulae in Mathematical Logics.

Skills Acquired:

Analytical Thinking: Linking different domains helps in developing analytical thinking.

Comprehension: The presentation is based on using Mathematic Logic theory and respective formula to solve real problems. To start the solution one should comprehend the problem.

Symbolic Computation: The significance of symbols used when working with Logic algebra.

Problem solving: Problems based on the understanding of properties of quantors are linked to theoretical information.

Use and application: Significance of Logic algebra for other domains is mentioned.

8. Decimal form of numbers: to be “huge” or not to be

Manual of Scripts for MATHeatre: page 48

Math Topic: Fractions and decimal numbers

Age Group: 9-13

Knowledge Background Needed: Decimal numbers, ordering decimal numbers, periodic numbers, and fractions.

Knowledge Acquired: Role of place value.

Skills Acquired:

Comprehension: Comprehension of decimal numbers and fractions is deepened.

Numerical and Symbolic Computation: Development of numerical computation with decimal numbers and fractions.

Communication (mathematics communication): Clear description of own thinking processes and defending own ideas and looking for arguments.

9. Equation: the tragedy of the unknown

Manual of Scripts for MATHeatre: page 50

Math Topic: Equations

Age Group: 14-18

Knowledge Background Needed: addition, subtraction, equation notion, and multiplication.

Knowledge Acquired: separation of the unknown from known numbers, division by the coefficient of the unknown, find the lowest common denominator (cancellation of denominators), and distributive property.

Skills Acquired:

Comprehension: understanding of different methods for solving equations.

Numerical and Symbolic Computation in Logic are developed.

Use and application: To attract low- achievers.

10. Euclid's dream

Manual of Scripts for MATHeatre: page 53

Math Topic: Operations

Age Group: 9-13

Knowledge Background Needed: addition, multiplication and division.

Knowledge Acquired: mathematical operations are important in life. (Re)- discovery of dividend, divisor, quotient and remainder.

Skills Acquired:

In personification of the different operations students understand that each of them is important and that are need to be used to solve problems. With humour students (re)discover the role of each of them. Students develop communication and mathematical demonstration.

11. A beauty Contest for Quadrilaterals

Manual of Scripts for MATHeatre: page 58

Math Topic: Geometry (plane figures)

Age Group: 14-18

Knowledge Background Needed: basic geometric figures: triangle, quadrilateral, rectangle, hexagon, circumscribed figures.

Knowledge Acquired: properties of basic plane geometry figures, connected with symmetry, circumscription and convexity.

Skills Acquired:

Analytical Thinking: Linking different properties requires the development of analytical thinking.

Visualization skills are developed, as graphical drawings are needed, in order to visualize properties and observations of the problems. Symmetry and convexity develops imagination.

Problem solving: Problems based on the understanding of properties of geometric figures linked to theoretical information.

Use and application: Significance of plane geometric figures for other domains is mentioned.

Communication: Preparing solutions of problems students use visual tools, which develops communication skills.

12. An one-act play for four operations

Manual of Scripts for MATHeatre: page 70

Math Topic: Operation with numbers and vectors

Age Group: 14-18

Knowledge Background Needed: Four numerical operations with numbers, description of basic theorems in the algebraic way, vector arithmetic.

Knowledge Acquired: Deepening of understanding the operations dealt with and of mutual similarities and differences.

Skills Acquired:

Comprehension: The understandings of the topics dealt with are: deepened, mutual links of different domains are developed, the mathematics behind them become more complicated without sufficient algorithmic comprehension.

Numerical and Symbolic Computation are needed for understanding the problem dealt with.

Use and application: Application of basic facts from one domain occurs in relationship with another domain. It is a less philosophical, more practical series of dialogues which aim to present the properties of the four basic operations,

Communication (mathematics communication): Description of concepts and formulation of properties is developed. The text seems to be a good drama, but contains some remarks which are less suitable for the age groups in our vision .

13. Percentages: the haughtiest of all fractions

Manual of Scripts for MATHeatre: page 78

Math Topic: Arithmetic, Decimal and Sexagesimal Numerals, Fractions, Percentages

Age Group: 9-13

Knowledge Background Needed: Work with fractions, percentages, denominators, and superabundant numbers.

Knowledge Acquired: History of sexagesimal and decimal fractions, there is no superior of fractions, percentages are clear information.

Skills Acquired:

The preparation and presentation required for this MATHeatre play develops Numerical and Symbolic Comprehension for pupils: the understanding of decimal and sexagesimal numbers and fractions, the use of superabundant numbers and the expression of fractions as percentages.

The students learn about the history of mathematics. They learn about sexagesimal numbers being the oldest system.

Numerical Computation is needed to understand the problem.

Visualization skills are developed as graphical drawing is required in order to visualize the mathematical solution and observation of the content.

Use and applicability: It can be seen that the use of youth language in maths brings lot of interest and high motivation to learn fractions and percentages. Fun in mathematics combined with learning is the main task of this play –it needs additional instruction to be understood. It is easy to use and can be rehearsed with all classes.

Preparing the problems, the presentation with the appropriate scenario and acting develops the Communication skills of the pupils.

14. Living down-town or in the suburbs? A hard question to answer…

Manual of Scripts for MATHeatre: page 81

Math Topic: Inscribed angles

Age Group: 14-18

Knowledge Background Needed: properties of circle.

Knowledge Acquired: inscribed angle theorem, obtuse angle, central angle, adjacent angles.

Skills Acquired:

Students discover a way of demonstration

Personification of angles, symbolic comprehension

Students learn to explain, make hypothesis and visualize geometry in space

15. The circle and the others

Manual of Scripts for MATHeatre: page 85

Math Topic: Geometry (polygons and circle)

Age Group: 14-18

Knowledge Background Needed: straight line, polygon, circle, central line and tangent.

Knowledge Acquired: A polygon tends to a circle when the number of vertices increases, idea of friction.

Skills Acquired:

Analytical Thinking: Linking different properties requires the development of analytical thinking.

Visualization skills are developed, as graphical drawing is needed, in order to visualize geometric properties

Use and application: Significance of tangent properties for other domains is mentioned.

16. The poor Thales becoming rich

Manual of Scripts for MATHeatre: page 88

Math Topic: History of Mathematics

Age Group: 9-13

Knowledge Background Needed: Knowing that Thales was a great Philosopher and Mathematician.

Knowledge Acquired: The insight that Philosophy and Mathematics are not abstract sciences but rather that they have a practical use for real life situations.

Skills Acquired:

The student first needs to collect information about Thales of Miletus. The History of Mathematics is the topic of this play.

A real life problem is solved using a mathematical solution. Learning mathematics brings advantages in real life is the message.

Use and applicability: It can be seen that the use of flexible thinking has always been and will continue to be most effective.

Preparing the problems, the presentation with the appropriate scenario, acting and the use of visual tools develops the Communication skills of the pupils.

17. A Number of Numbers

Manual of Scripts for MATHeatre: page 94

Math Topic: Math in everyday life, Fibonacci, Golden ratio

Age Group: 9-13

Knowledge Background Needed: Some properties of numbers.

Knowledge Acquired: relevance of mathematics with everyday concepts, the Golden ratio and Fibonacci sequence in real objects, math history.

Skills Acquired:

Visualization: math in everyday objects and numbering.

Communication (mathematics communication): math in everyday life, introductory number series and geometry concepts, relevance with everyday life.

18. Political Numbers

Manual of Scripts for MATHeatre: page 109

Math Topic: geometrical progression

Age Group: 14-18

Knowledge Background Needed: money and cent multiplication.

Knowledge Acquired: mathematical properties of geometry progression of numbers.

Skills Acquired:

Through a concrete situation in a conceived government, student understands the properties of calculation.

In such case, student can approach mathematics with a concrete attractive story with a little understanding of dark humour!

19. “distant.relations”

Manual of Scripts for MATHeatre: page 113

Math Topic: Distances between the planets

Age Group: 14-18

Knowledge Background Needed: distance, ratio, basic facts from Astronomy concerning the planets of the Solar system.

Knowledge Acquired: relativity of distances.

Skills Acquired:

Analytical Thinking: Linking different domains requires the development of analytical thinking.

Numerical Computation: approximations in computing of big numbers.

Use and application: Significance of distances and ratio for other domains, Astronomy included.

20. Noname

Manual of Scripts for MATHeatre: page 118

Math Topic: Basic computations

Age Group: 9-13

About the script: The principal character is going through different enigmas all along the story; enigmas are of mathematical nature and refer to real life problems. The answers are not given in the script, so one can then assume that it's up to the audience in class to answer together, which makes this play an interactive one.

Knowledge Background Needed: basic knowledge about addition, division, subtraction, multiplication.

Knowledge Acquired: numerical calculation, mental computation (counting 5 from 5), time calculation, odd numbers and even numbers.

Skills Acquired:

Comprehension: logical reasoning.

The pupils deepen their skills in computation through mathematical enigmas.

Use and application: This type of script can be used to improve every different topics the teacher wants to teach, he just have to adapt the enigmas. It's a funny way for the pupils to practice.

21. Beyond Infinity

Manual of Scripts for MATHeatre: page 123

Math Topic: Arithmetical reflections on infinitive numbers, the gap between "school mathematics" and "problem solving".

Age Group: 14-18

Knowledge background: Real life experience in mathematics lessons based on the traditional syllabus; basic knowledge of arithmetic; infinitive numbers.

Knowledge Acquired: Infinitive number problems (addition and subtraction of infinitive numbers). Knowledge, that Ada is an object-orientated high level computer programming language, developed from Pascal. Ada was named after Lady Ada Lovelace (1815-1852) who was the first computer programmer.

Skills Acquired:

The preparation and presentation required for this MATHeatre play develops Numerical and Symbolic Comprehension for pupils: the understanding of infinitive numbers – the possibility to add them and the problem of subtraction.

The students learn about the history of mathematics. They learn that the computer language Ada was named after Lady Ada Lovelace.

Numerical and Symbolic Computation is needed to understand the problem.

They learn that Problem solving is an important part of Mathematics and that "school mathematics" does not cover all important mathematical problems.

Use and applicability: It can be seen that the use of youth language and responding to school problems in maths causes a lot of interest plus a lot of motivation to solve problems.

Preparing the problems, the presentation with the appropriate scenario and acting develops the Communication skills of the pupils.

22. Math Homework

Manual of Scripts for MATHeatre: page 130

Math Topic: Everyday mathematics.

Age Group: 9-13

Knowledge Background Needed: simple operations, introductory sets.

Knowledge Acquired: mathematics in everyday life, mathematical thinking, and math history.

Skills Acquired:

Communication (mathematics communication): math history, math in everyday life problems.

23. The four guardians of the scared philosopher

Manual of Scripts for MATHeatre: page 133

Math Topic: Numbers

Age Group: 9-13

Knowledge Background Needed: knowledge about numbers.

Knowledge Acquired: understanding the vital role of the zero, definition of prime numbers, information about numerical system, realize the importance of numbers existence, definition of irrational numbers.

Skills Acquired:

Comprehension: logical arguing.

The students learn about the history of mathematics. They also learn about the discovery of the numbers.

Use and application: To develop pupils' curiosity.

24. The Chronicles of Catherine Cloud

Manual of Scripts for MATHeatre: page 139

Math Topic: Pythagoras and numbers

Age Group: 9-13

Knowledge Background Needed: ideas about numbers, shapes, circumference of the circle, radius, Pi.

Knowledge Acquired: mathematical notions around circle: tangents, secants, chords.

Student develops mathematical knowledge through visiting different time periods.

Skills Acquired:

In personification of the different uses of mathematic in life students understand that it is important and that we need to use them to solve problems: each geometric figure has its own properties to apply in concrete cases.

With humour students (re)discover the role of each mathematical discovery like numbers- history of mathematical notions.

25. The trial of numbers

Manual of Scripts for MATHeatre: page 139

Math Topic: Numbers

Age Group: 14-18

Knowledge Background Needed: integers, zero, rational and irrational numbers, infinity.

Knowledge Acquired: the necessity of introducing irrational numbers.

Skills Acquired:

Analytical Thinking: Linking different properties requires the development of analytical thinking, why it is not allowed to divide by zero (thus going to infinity).

Comprehension: The historical reasons for introducing irrational numbers help to understand the importance of the irrational numbers.

Numerical Computation: The significance of the irrational numbers to computation is shown.

Use and application: Significance of the zero, infinity and the irrational numbers for other domains is mentioned.

26. "Conditions, Conditions"

Manual of Scripts for MATHeatre: page 154

Math Topic: Quantifiers, logic

Age Group: 14-18

Knowledge Background Needed: Basics of mathematics logic.

Knowledge Acquired: Deeper insight in the properties of quantifiers.

Skills Acquired:

Analytical Thinking: Deeper insight in the properties of quantifiers.

Comprehension: This part of mathematical logic has important applications not only in mathematics, but also in everyday situations.

Problem solving: Application of mathematics concepts and their properties. The story is well constructed, has relation to mathematical content.

Use and application: Examples of the use of mathematical concepts and their application in various, real life-like situations applied to the correct definitions in logics.

Communication (mathematics communication): The clear description of concepts and their properties is developed, concerning its form it is more a stand-up comic-tragedy.

27. A unique ride

Manual of Scripts for MATHeatre: page 156

Math Topic: Numbers (proportions)

Age Group: 9-13

Knowledge Background Needed: Word tasks on proportions.

Knowledge Acquired: methodology in the solution of word tasks on proportions by ratio per unit.

Skills Acquired:

Analytical Thinking: Linking different domains requires the development of analytical thinking.

Mathematical modelling: skills to translate real life problems to mathematical problems, to find the corresponding mathematical solutions and to make the inverse translations the real life situation. All these stages are implemented and therefore mathematical modelling skills acquisition is supported.

Use and application: Significance of word mathematical tasks for other domains. Using money in an amusement park each student argues to convince the others. The entertainment way of presenting is a motivation to successful learning.

28. Elf numbers

Manual of Scripts for MATHeatre: page 162

Math Topic: Basic properties and writing of natural numbers, history of mathematics

Age Group: 9-13

Knowledge Background Needed: Basic properties of natural numbers, their notation in different cultures and the History of the calculations are needed.

Knowledge Acquired: Deepening of understanding the properties of systems used in writing the numbers and notations of the basic operations in different cultures.

Skills Acquired:

Comprehension: The understanding of the notations dealt with are deepened, mutual links of different domains like history of mathematics in different cultures are developed.

Numerical and Symbolic Computation for elementary calculations and properties of the natural numbers.

Use and application: The story of the author helps a deeper mathematical understanding, and the presentation is suitable for increasing the real understanding of history of numbers, the intercultural aspects are present by the personages appearing: an Egyptian, an Indian, a Roman and a Greek are arguing for their mathematical culture.

Communication (mathematics communication): Description of numbers and notations used to represent them is developed in a very original way, a fairy tale about a fictive person called Elf, and introducing the main character, Andrew to the history of numbers throughout thousands of years.

29. The fastest proof of everything

Manual of Scripts for MATHeatre: page 166

Math Topic: Pythagorean Theorem, proof, logic, language of mathematics

Age Group: 14-18

Knowledge Background Needed: Different parts of mathematics, logic and history of science.

Knowledge Acquired: Language of logic, symbols and mathematics.

Skills Acquired:

The preparation and presentation required for this MATHeatre play develops Symbolic Comprehension for pupils: the understanding of different symbols (not only from mathematics). The student also learns about the history of mathematics.

They learn that Problem solving is an important part of Mathematics and the proof is the basis of mathematical thinking.

Preparing the problems, the presentation with the appropriate scenario and acting develops the Communication skills of the pupils.

30. Mathsss... Puaghh...!!! What for?

Manual of Scripts for MATHeatre: page 171

Math Topic: Golden Ratio

Age Group: 9-13

Knowledge Background Needed: basic knowledge about addition, division.

Knowledge Acquired: Golden Ratio, deduction.

Skills Acquired:

Comprehension: logical reasoning. The students learn about the golden number

Use and application: To develop pupils' curiosity.The presentation is suitable for increasing the pupils' curiosity and to make them change their mind about mathematics.

31. Circles, semicircles and math

Manual of Scripts for MATHeatre: page 175

Math Topic: Logarithms

Age Group: 14-18

Knowledge Background Needed: Archimedes, Pythagoras, Logarithm.

Knowledge Acquired: History of this men and of logarithm. How it's used today concretely (logarithm).

Skills Acquired:

In personification of the different mathematicians students discover a way of demonstration. With humour students (re)discover the role of each mathematician. Students learn to explain and change their attitude towards mathematics.

32. Around the circle

Manual of Scripts for MATHeatre: page 178

Math Topic: Geometry

Age Group: 9-13

Knowledge Background Needed: Basic properties of geometry.

Knowledge Acquired: Learning the calculation of perimeter and area of basic plane figures with emphasis on circle.

Skills Acquired:

Relating games with geometry figures using reflective modern ideas.

33. Monkey Business

Manual of Scripts for MATHeatre: page 187

Math Topic: Numbers

Age Group: 9-13

Knowledge Background Needed: multiplication and division of integers, divisor, and multiplier.

Knowledge Acquired: skills to find LCM (least common multiplier).

Skills Acquired:

Analytical Thinking: Linking different properties requires the development of analytical thinking.

Numerical computation: skills for mental computation

Problem solving: Problems based on the understanding of properties of numbers are linked to theoretical information. Skills to transform real life problems to mathematical problems,, to find the corresponding mathematical solutions and to make the inverse translations in the real life situation.

Use and application: Significance of LCM for other domains is mentioned. The problem is developed as an enigma, which increases curiosity and is a motivation to learning.

34. The Pythagorean proposition

Manual of Scripts for MATHeatre: page 199

Math Topic: The goal of this act is to be teached the Pythagorean Proposition and its reverse through one practical problem. The script clearly states the actuality: a difficulty in drawing the right angles and the goal.

Age Group: 14-18

Knowledge Background Needed: Description of basic theorems in elementary number theory, and Pythagoras' theorem, the History of the calculations are needed.

Knowledge Acquired: Deepening of understanding the applicability of school mathematics.

Skills Acquired:

Comprehension: The understanding of the topics dealt with is deepened, mutual links of different domains like history of mathematics, theoretical and practical computation aspects are developed.

Numerical and Symbolic Computation: calculations and properties of the natural numbers and applications of Pythagoras' theorem.

Use and application: a deep mathematical understanding and the presentation is suitable for increasing the real understanding of real applied mathematics.

Communication: creating a real dramatic situation around the relation between the personages help to develop good communication skills.

35. A mathematician's Apology

Manual of Scripts for MATHeatre: page 210

Math Topic: 3D geometry

Age Group: 14-18

Knowledge Background Needed: History and discovery.

Knowledge Acquired: Reflexion about mathematics in our world. How it's used today concretely: puzzles, numbers, in poetic and in painting.

Skills Acquired:

Students discover a way of demonstration through humour the role of each mathematical application. Students learn to explain, make hypothesis and change their attitude toward mathematics.

36. Operation: Equation

Manual of Scripts for MATHeatre: page 219

Math Topic: Algebra

Age Group: 9-13

Knowledge Background Needed: Properties of arithmetic.

Knowledge Acquired: Apply properties of arithmetic with emphasis in the order of operations and progressions.

Skills Acquired:

The script is helping the pupils to develop a broad range of skills such as the knowledge of applications, communication and collaboration, self-direction, motivation and learning how to learn. It creates the environment for reflection and comprehension of concepts and processes around this mathematical area.

37. The happiness scale and the history of imaginary numbers

Manual of Scripts for MATHeatre: page 224

Math Topic: Number sets with the focus mainly on complex numbers.

Age Group: 14-18

Knowledge Background Needed: Work with numbers sets, especially focusing on complex numbers.

Knowledge Acquired: Historical development of number sets, deepening of knowledge about properties of numbers.

Skills Acquired:

Analytical Thinking: Linking different domains requires the development of analytical thinking.

Comprehension: The historical reasons for introducing complex numbers are one of tools helping to understand the importance and properties of complex numbers.

Numerical and Symbolic Computation: The significance of symbols used when working with complex numbers is shown.

Problem solving: Problems based on the understanding of properties of numbers are linked to theoretical information.

Use and application: Significance of complex numbers for other domains is mentioned.

38. On the set of the movie "How to become a Pythagorean"

Manual of Scripts for MATHeatre: page 224

Math Topic: History of Mathematics, popularization of Mathematics

Age Group: 14-18

Knowledge Background Needed: The History of Pythagoras' theorem, and film making.

Knowledge Acquired: Better understanding of the Pythagoras Theorem.

Skills Acquired:

Comprehension: The understanding of the topics dealt with are deepened, mutual links of different domains like history of mathematics in different cultures are developed.

Numerical and Symbolic skills: Formulation and calculations related to Pythagoras' theorem

Use and application: The story invented by the author helps to understand the real life vocabulary of the world of making films, as a work-film about the subject

Communication (mathematics communication): The short film scenario about the subject formulated in the title, suitable for a larger audience – like advertising clip about the project.

39. Who is better?

Manual of Scripts for MATHeatre: page 232

Math Topic: trigonometry functions

Age Group: 9-13

Knowledge Background Needed: introductory trigonometry, functions.

Knowledge Acquired: relation of trig functions.

Skills Acquired:

Numerical and Symbolic Computation: relation of trig functions, absolute values, Cartesian coordinate system.

Visualization: relation of trig functions.

Communication (mathematics communication): functions appear as characters connected by their relations.

ANNEX 2 - Mathematical Stories for Theatre Analysis

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Page

1. Elementary Operations

The children at Santa's village

Math Topic: Arithmetic

Age Group: 9-13

Knowledge Background Needed: No special knowledge background is required for a child to fully comprehend this story.

Knowledge Acquired: Mathematical operations: addition, subtraction, multiplication, division.

Skills Acquired:

This story develops in the most vivid way the comprehension skills of the students, as it uses the same example with the gift boxes to present a step-by-step description of the four mathematical operations. Taking advantage of the positive feelings Christmas and Santa Clause themes bring to kids, it presents addition, subtraction, multiplication and division in a way students are able to fully understand and follow. Moreover, it uses story-telling and narration as tools for mathematics communication. Finally, use and application of basic arithmetic in a production line is also present in this story.

2. Straight lines and angles

Trupot the robot learns straight lines and angles

Math Topic: Geometry

Age Group: 9-13

Knowledge Background Needed: Circle, rectangle, measuring angles, radius and diameter.

Knowledge Acquired: Differentiate and define line, ray and segment. Define and classify angles (acute, right, and obtuse).

Skills Acquired:

Analytical thinking skills: Description of motion using geometric concept of a straight line.

Understanding: Relationship between the ideas of infinity, beginning and ending with the definitions of line, ray and segment.

Numerical and Symbolic Computation: The "greater than" and "less than" operators are handled.

Problem solving skills: problem is described and its solution presented.

Mathematical modeling skills: a real situation is described with a mathematical model (straight-line trajectory) (segment-start and end).

Visualization skills: Development of the geometric view, locate and describe an environment full of geometric shapes, 3D viewing angles.

Use and applicability: spatial concepts that allow us to interpret, to understand and to appreciate the environment.

Communication skills: appropriate use of mathematical language.

3. Triangles

In the land of mathematic triangles

Math Topic: Geometry. Teach young students the fundamental notions regarding triangles. More precisely, their classification according to sides and angles.

Age Group: 9-13

Knowledge Background Needed: an easy to read story while enables students to understand and identify triangles according to two criteria: classification by sides and by angles.

Knowledge Acquired: 'Triangles' uses a simple scenario to stimulate the acquisition of new knowledge through the understanding of the mathematical motions regarding geometrical forms.

Knowledge Acquired: Students learn about the equilateral, isosceles and the scalene triangles, as well as about the acute, obtuse and the rectangle triangles.

Skills Acquired:

Use and applicability: The simple, real to life language is to arouse both interest and motivation towards learning about the world of Mathematics in general, that of the triangles in particular.

Students may thus understand that each triangle is different and has no connection with any of the triangles presented in the scene.

4. Plane Shapes

Sophie at the land of plane shapes

Math Topic: Geometry

Age Group: 9-13

Knowledge Background Needed: No special knowledge background is required for a child to fully comprehend this story.

Knowledge Acquired: Plane shapes, squares, rhombus, trapeziums, triangles, rectangulars, rectangular parallelograms, circles, polygons.

Skills Acquired:

This story gives a presentation of the various plane shapes by stimulating imagination and describing a journey to the land of plane shapes. It boosts comprehension skills by presenting beautiful images and metaphors. If presented the way written, it has the potential of developing visualization skills by showing the differences between different shapes (angles, parallel lines etc.). The way the story is structured is also a nice example of mathematics communication, using a well-known story-telling trick (visiting an exotic land) to make math more attractive.

5. Curves

Curves at the Luna Park

Math Topic: Curves

Age Group: 9-13

Description of the story: The story concerns the visit of a class of students to the Luna Park and the identification in this context of a number of curves that can be exploited in order to help them understand the concept.

Knowledge Background Needed: No special knowledge background is required.

Knowledge Acquired: Understanding of curves.

Skills Acquired:

Relating real life applications to mathematics.

Useful approach in creating the momentum for studying curves. The story is helping the pupils in developing skills such as knowledge of applications, communication and collaboration, self-direction, motivation and learning how to learn. It creates the environment for reflection and comprehension of concepts and processes around this mathematical area.

6. Perimeter-Area

The measure-area

Math Topic: Perimeter- Area, The measure-area

Age Group: 9-13

Knowledge Background Needed: Square, rectangle, Rhombus, parallelogram, triangle, Area, Perimeter.

Knowledge Acquired: Formulas of Area and perimeter of a square, parallelogram, triangle, Rhombus, rectangle.

Skills Acquired:

The story boosts comprehension skills on how to calculate the area and perimeter of a triangle and the various types of parallelograms. Numerical and Symbolic Computation is mentioned when multiplying the area of a pillow which is 30cm^2 by 12 to get the area covered by the tent. Mind Visualization of all the shapes mentioned. There are no actual drawings however some of the shapes are described in a way that the student is able to recall the shape in his mind. Preparing the presentation with the appropriate scenario, and the acting develops the Communication skills of the pupils.

7. Sets

The most beautiful camping of the mathematicians

Math Topic: Sets (preliminary definitions from the Set Theory)

Age Group: 9-13

Knowledge Background Needed: simple reasoning.

Knowledge Acquired: definitions of set, subset, element of a set, inclusion, union of sets, and intersection of sets.

Skills Acquired

Analytical Thinking: finding inclusion, union, intersection.

Comprehension: knowing how to denote sets, union and intersection; mathematical modeling.

Problem solving: starting to solve the problem one should comprehend the problem and plan the solution.

Communication: skill of finding and presenting a mathematical idea (mathematics communication).

8. The Cube

The water cube

Math Topic: The Cube elements, Cube Volume

Age Group: 9-13

Knowledge Background Needed: Square, base, mass, length.

Knowledge Acquired: Volume of the cube, Number of edges, Cube diagonal, Angles on a Cube.

Skills Acquired:

The story enhances the comprehension skills on how to calculate the Volume of a cube. Mind Visualization of all the shapes mentioned. There are no actual drawings however some of the shapes are described in a way that the student is able to recall the shape in his mind. Preparing the presentation with the appropriate scenario, and the acting develops the Communication skills of the pupils.

9. The sphere

A sphere of other dimensions

Math Topic: Geometry

Age Group: 14-18

Description of the story: The story concerns a discussion between two children about the concept of dimension and a visit of the two to a utopian space. This gives them the opportunity to consider some concepts that constitute a space somehow different from the one they experience in everyday life. Also it provides opportunities for considering ideal conditions and for living and moral aspects that can be set as values.

Knowledge Background Needed: Basic mathematics.

Knowledge Acquired: The setting in which the story takes place contributes effectively in the comprehension of the concept of dimension and space.

Skills Acquired:

Useful approach in creating the momentum for studying elements of geometry that are not usually the object of school mathematics. The story is helping the pupils to develop skills such as knowledge of applications, communication and collaboration, self-direction, motivation and learning how to learn. It creates the environment for reflection and comprehension of concepts and processes around this mathematical area. Furthermore it provides the opportunity for values education.

10. The cone

The cone and Nic's construction

Math Topic: Basic properties of conic surfaces, central axes, semi-straight lines, vertex, circular basis, right cone, oblique cone, computer graphics

Age Group: 9-13

Knowledge Background Needed: Elementary space Geometry, points, angles, semi-lines, surface.

Knowledge Acquired: the notion of the conic surface, right cone, oblique cone, circular disc, elliptic disc, and cone shaped forms in everyday form.

Skills Acquired:

Comprehension: The understanding of the geometric construction and properties is deepened; links of different applications of cons in real life are developed.

Numerical and Symbolic Computation for graphing conical surfaces are developed.

Use and application: The play is increases the motivation of pupils towards learning mathematics; the story invented by the author helps the pupils find relations between the mathematics lesson and real life.

Communication (mathematics communication): an imaginary dialogue is developed between the teacher and pupils, and the ideas are continued in designing a game and competition based on the mathematics learned in the lesson, to increase the results to be obtained by pupils in the classroom of the main character. The logo of the story is "Knowledge is power".

11. The cylinder

The small Eskimo and the cylinder

Math Topic: cylinder: description of the solid and its volume

Age Group: 9-13

Knowledge Background Needed: Basic geometrical vocabulary: cylinder, surface, circle, radius, height.

Knowledge Acquired: This fairytale does include some basic information about cylinder. On using this play the theory is taught. It is possible to add other type of solids.

Skills Acquired:

The pupils realize that the mathematical knowledge can be needed in other fields than mathematics, that everyday life problems can be solved thanks to mathematics.

Use and applicability: It can be seen that using a fairytale is motivating and creates interest in a very abstract algebraic problem.

Preparing the problems, the presentation with the appropriate scenario, acting and the use of visual tools develops the Communication skills of the pupils.

12. Pyramid

The spatial pyramid

Math Topic: pyramid - description of the shape

Age Group: 9-13

Knowledge Background Needed: Basic geometrical vocabulary: pyramid, base, polygon, side, distance.

Knowledge Acquired: More special vocabulary: vertex, edge, height. This fairytale does include some basic information about pyramid. On using this play the theory is taught. It is possible to add other type of solids.

Skills Acquired:

The pupils realize that the mathematical knowledge can be needed in other fields than mathematics, that everyday life problems can be solved thanks to mathematics.

Use and applicability: Using this type of fairytale is a way to motivate pupils, creating interest around mathematical notions. Others different mathematical shapes could be added in the script to discover or describe other solids that the teacher needs to teach regarding to the curriculum.

Preparing the problems, the presentation with the appropriate scenario, acting and the use of visual tools develops the Communication Skills of the pupils.

13. Prism

A meteor prism

Math Topic: Geometry, Stereometry, prism, crystals

Age Group: 9-13

Knowledge Background Needed: prism.

Knowledge Acquired: Terminology connected with prism.

Skills Acquired:

Use and applicability: nice example of the use of mathematical terminology in real life situation. Crystals are examples of perfect prisms.

Preparing the problems, the presentation with the appropriate scenario and acting develops the Communication skills of the pupils. It is very important, that theatre play shows the correct terminology.

14. Equal Triangles-Uneven relations

A different lesson

Math Topic: Congruence of triangles.

Age Group: 9-13

Knowledge Background Needed need: basic knowledge of properties of triangles.

Knowledge Acquired: Deepening the knowledge of the congruence of triangles, above all the application of the three basic theorems (Side-Side-Side, Side-Angle-Side, Angle-Side-Angle, Angle-Angle-Side) in various situations and assigned elements of triangles. Application for right-angled triangles.

Skills Acquired:

Improving communication skills by being in the position requiring explanations of mathematical ideas.

Improving the competency to pose question and to defend own ideas.

15. Pythagoras' theorem

Ancient, Greek, Mathematical museum

Math Topic: Pythagoras' theorem

Age Group: 9-13

Knowledge Background Needed: Right-angled triangles, Pythagoras' theorem.

Knowledge Acquired: The names of famous ancient mathematicians are mentioned. This fairytale explains the mathematical content of Pythagoras' theorem. On using this play the theory is taught.

Skills Acquired:

Use and applicability: Using this type of fairytale is a way to motivate pupils and to create interest around mathematical notions.

Preparing the problems, the presentation with the appropriate scenario, acting and the use of visual tools develops the Communication skills of the pupils.

16. Longitude and latitude and international time

A birthday present

Math Topic: Geometry, Planet rotation & Time (time-zones)

Age Group: 9-13

Knowledge Background Needed: Basic mathematics.

Knowledge Acquired: Learning about the Earth's rotation, how it effects time, and the division of 24 time-zones. This fairytale does include some basic information about the earth moving around its own axle over 24 hours.

Skills Acquired:

Problem solution skills using a mathematical solution. Mastering the earths division in 24 time-zones and reflecting the time of day and night.

Use and applicability: It can be seen that using a fairytale is motivating and creates interest in a large geographical object as the Earth and the construction of time in days and hours after its rotation around its own axle.

Preparing the problems, the presentation with the appropriate scenario, acting and the use of visual tools develops the Communication skills of the pupils.

17. Factorial

The puzzle of knowledge of the green dragon

Math Topic: Factors, combined mathematics

Age Group: 9-13

Knowledge Background Needed: basic mathematics.

Knowledge Acquired: Basic information about factors. Understanding of factorial.

Skills Acquired:

Problem solving skills supported by mathematical solution. To learn factors can be seen as being an advantage and achieving success.

Use and applicability: It can be seen that using a fairytale is motivating and creates interest in a very abstract algebraic problem.

Preparing the problems, the presentation with the appropriate scenario, acting and the use of visual tools develops the Communication skills of the pupils.

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